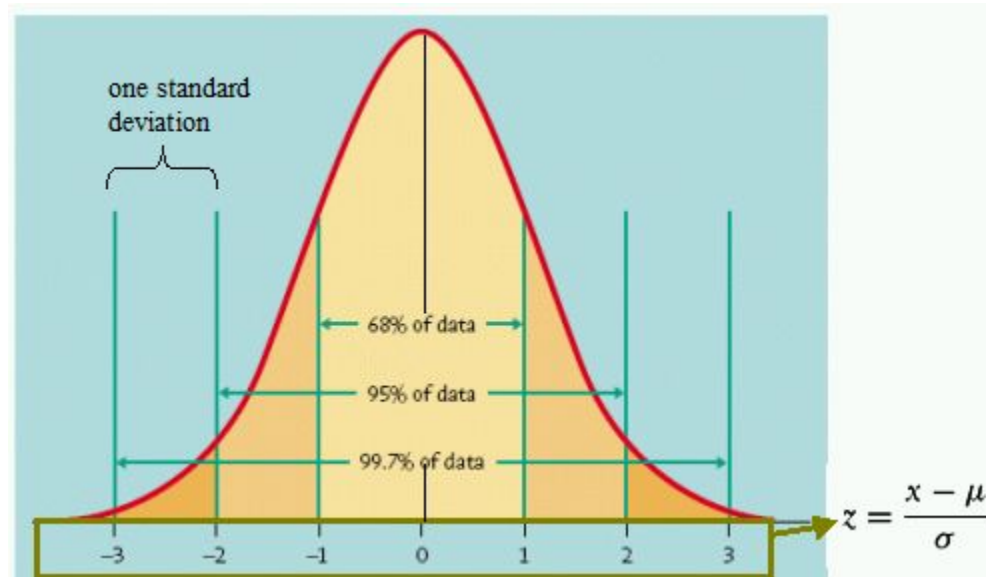


THE NORMAL DISTRIBUTION



We've already seen a few attempts to define what is known as the *normal distribution*.

We've already seen a few attempts to define what is known as the *normal distribution*.

1. A bell-shaped distribution that is symmetrical.

We've already seen a few attempts to define what is known as the *normal distribution*.

1. A bell-shaped distribution that is symmetrical.
2. A symmetrical bell-shaped distribution that has 68% of its scores within 1 standard deviation of the mean, 95% within 2 standard deviations, and 99.7% within 3 standard deviations.

And now we are ready for the definitive definition.

1. A bell-shaped distribution that is symmetrical.
2. A symmetrical bell-shaped distribution that has 68% of its scores within 1 standard deviation of the mean, 95% within 2 standard deviations, and 99.7% within 3 standard deviations.

And now we are ready for the definitive definition.

1. A bell-shaped distribution that is symmetrical.
2. A symmetrical bell-shaped distribution that has 68% of its scores within 1 standard deviation of the mean, 95% within 2 standard deviations, and 99.7% within 3 standard deviations.
3. A normal distribution a function of the form:

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

In this formula, *mu* is the mean of the distribution and *sigma* is the standard deviation

$$y = f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

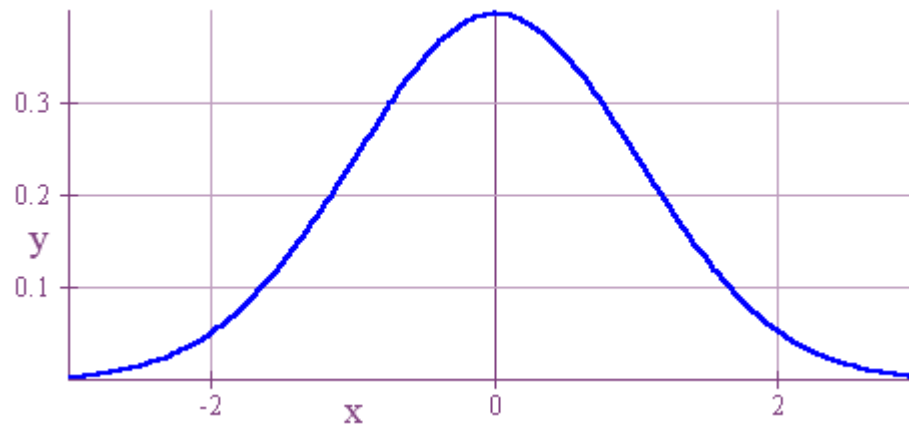
The normal distribution with $\mu = 0$ and $\sigma = 1$ is called the *standard normal distribution*.

$$y = f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$



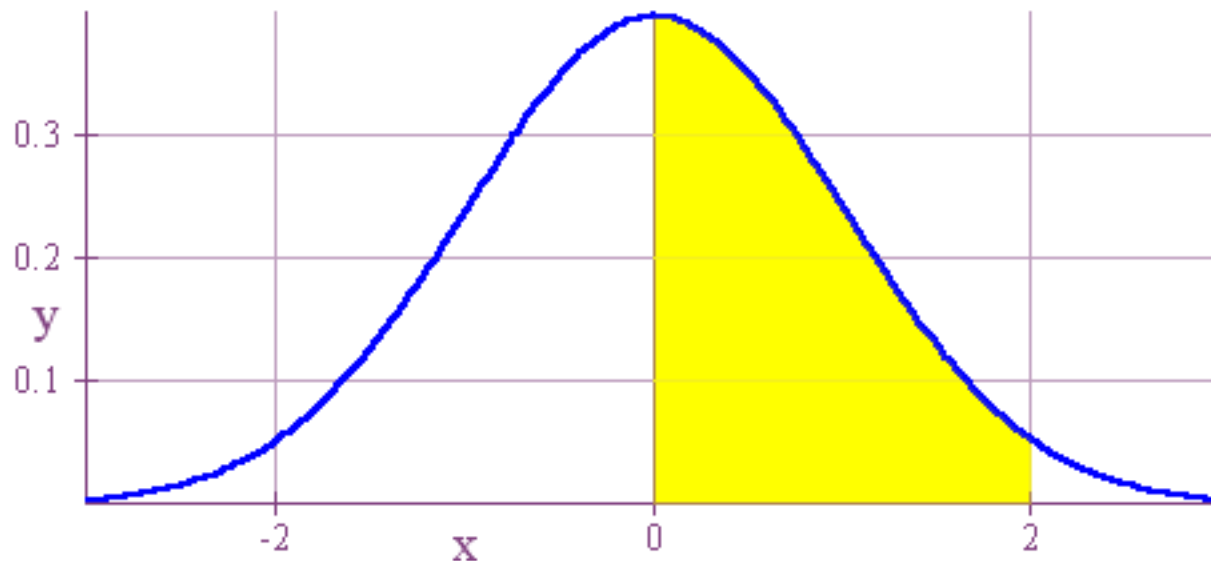
For all normal distributions, the area under the curve is equal to 1.

$$y = f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$



To find the probability that an x -value below is between zero and two, we just need to know the area under the curve from zero to two.

$$y = f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$



Here are some examples of finding probabilities using a standard normal distribution (*mean=0, standard deviation=1*).

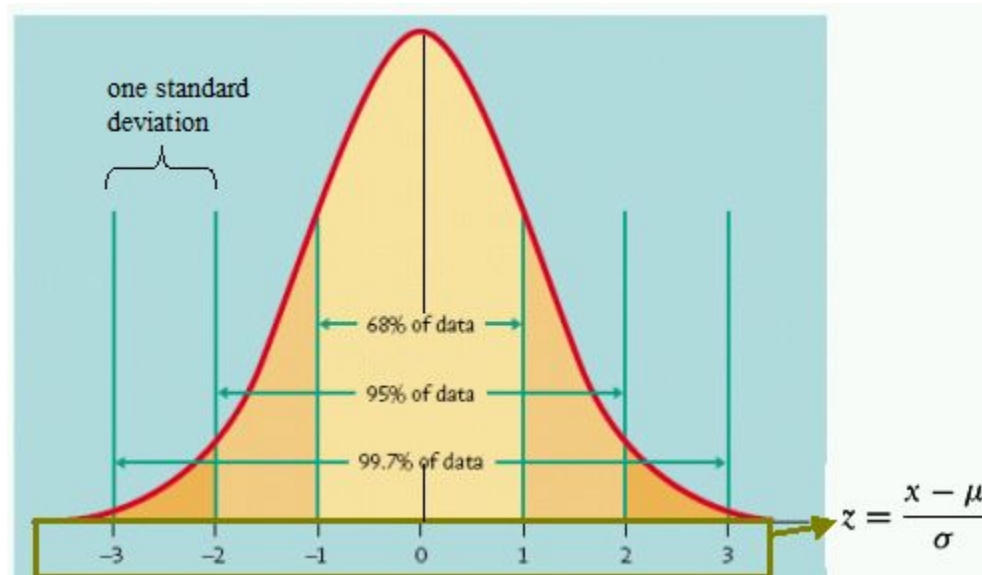
The probability that a score is within 1 standard deviation of the mean.

Here are some examples of finding probabilities using a standard normal distribution (*mean=0, standard deviation=1*).

The probability that a score is within 1 standard deviation of the mean.

```
normalcdf(-1,1)  
.6826894809
```

$$P(-1 \leq z \leq 1)$$

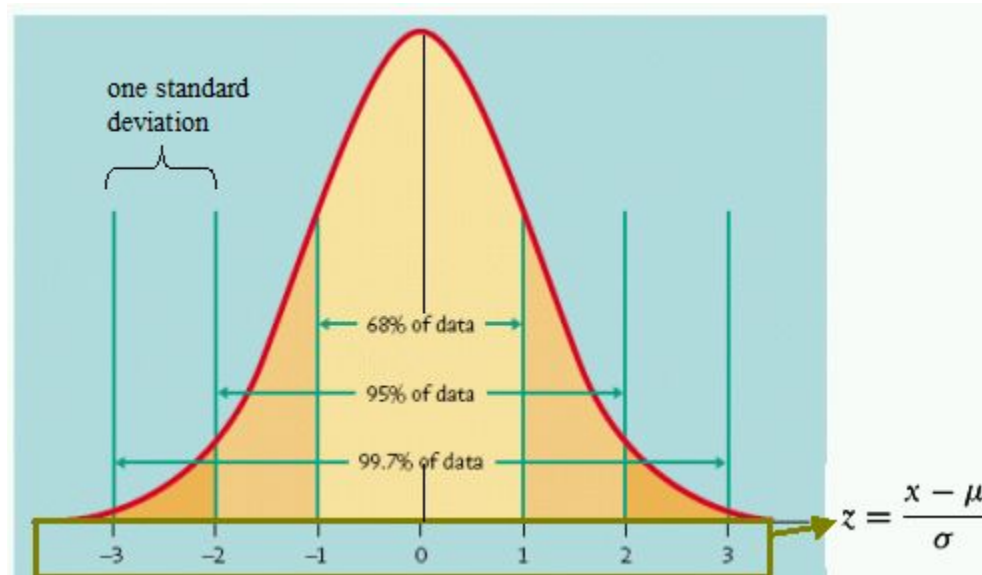


Here are some examples of finding probabilities using a standard normal distribution (*mean=0, standard deviation=1*).

The probability that a score is within 2 standard deviations of the mean.

```
normalcdf(-2,2)  
.954499876
```

$$P(-2 \leq z \leq 2)$$

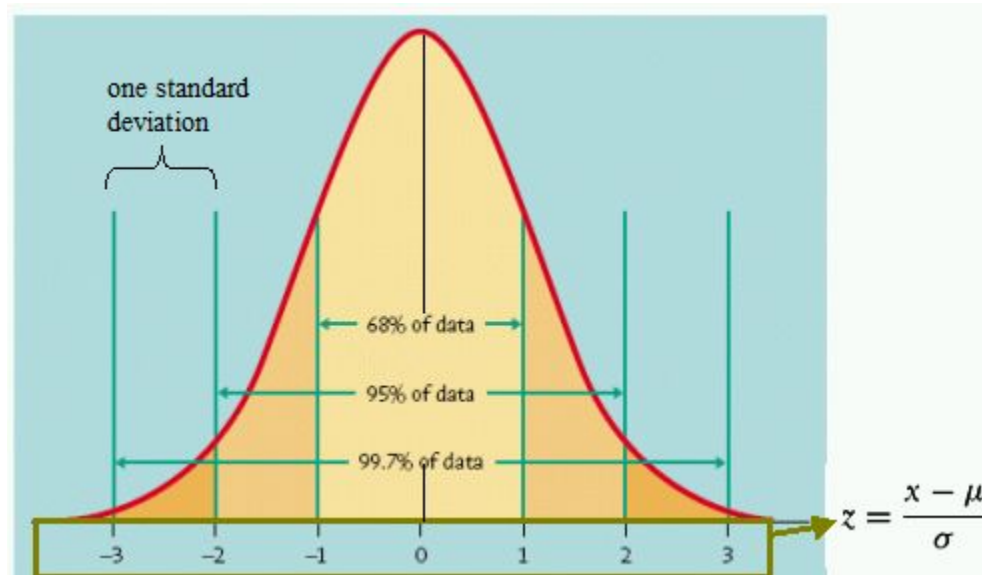


Here are some examples of finding probabilities using a standard normal distribution (*mean=0, standard deviation=1*).

The probability that a score is within 3 standard deviations of the mean.

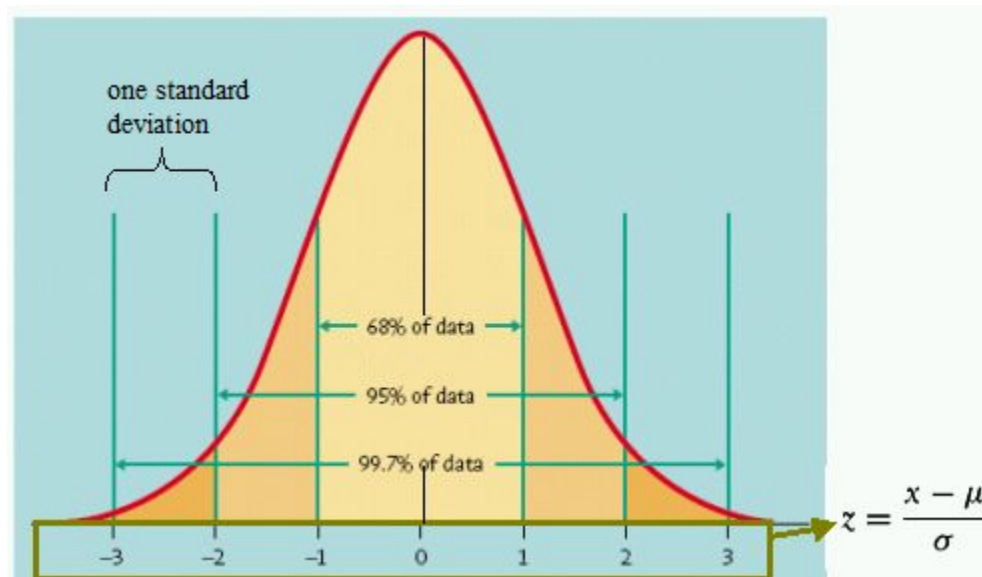
```
normalcdf(-3,3)  
.9973000656
```

$$P(-3 \leq z \leq 3)$$



What if we want to find the probability that a score's deviation from the mean is greater than 1 (*mean=0, standard deviation=1*)?

$$P(z > 1)$$

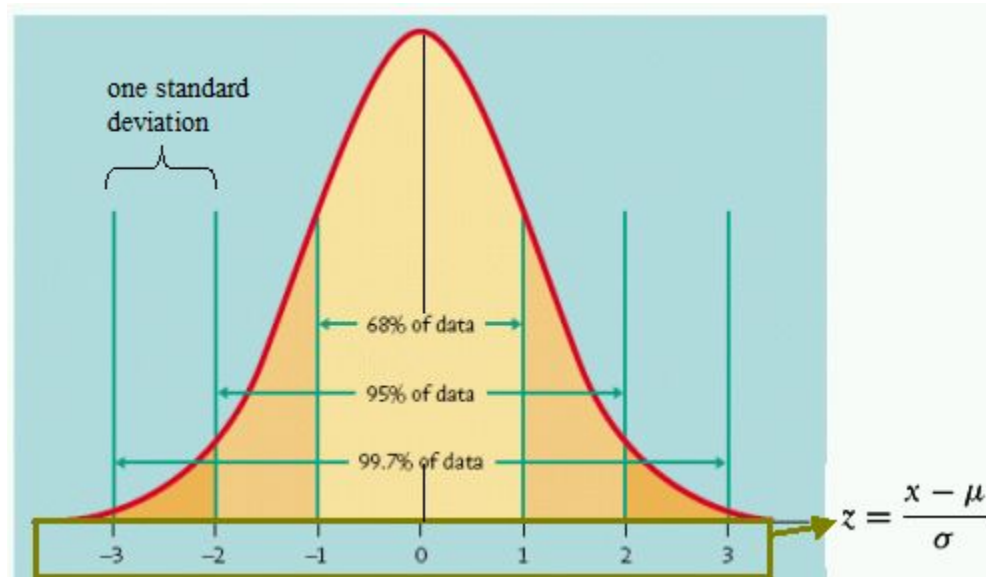


What if we want to find the probability that a score's deviation from the mean is greater than 1 (*mean=0, standard deviation=1*)?

We could do it two ways. The less accurate but good enough way is illustrated below.

```
normalcdf(1,9999  
99)  
    .1586552596
```

$P(z > 1)$

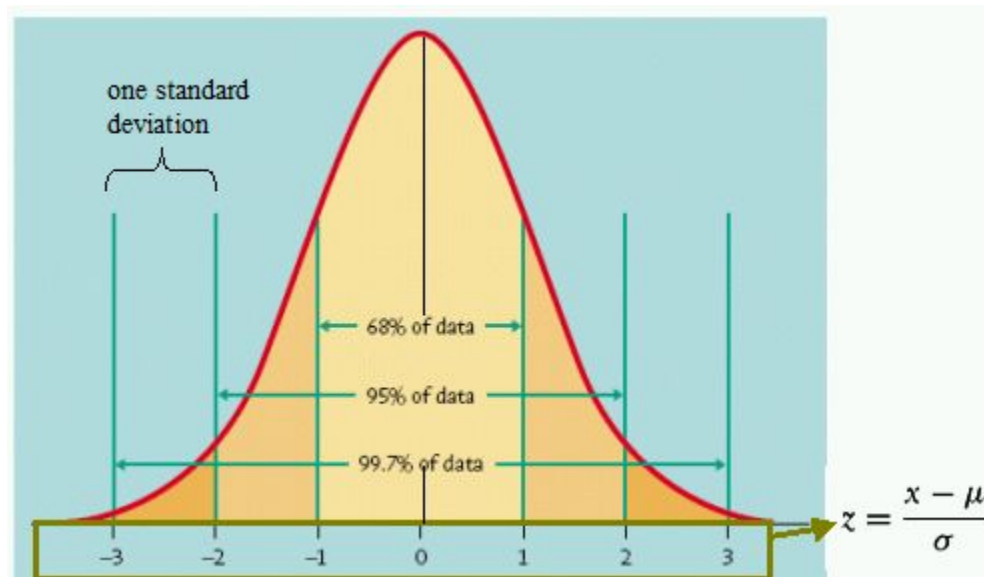


What if we want to find the probability that a score's deviation from the mean is greater than 1 (*mean=0, standard deviation=1*)?

And this is followed by the more accurate method.

$$P(z > 1)$$

```
normalcdf(1,9999  
99)  
.1586552596  
1-.5-normalcdf(0  
,1)  
.1586552601
```



What is we want to find areas or probabilities with respect to some other normal distribution where the *mean* is not zero and the *standard deviation* is not one?

What is we want to find areas or probabilities with respect to some other normal distribution where the *mean* is not zero and the *standard deviation* is not one?

In that case follow the syntax *normalcdf(start,stop,mean,deviation)*

What is we want to find areas or probabilities with respect to some other normal distribution where the *mean* is not zero and the *standard deviation* is not one?

In that case follow the syntax *normalcdf(start,stop,mean,deviation)*

```
normalcdf(85,115  
,100,15)  
    .6826894809
```

Another useful tool is *inverse normal* tool where we input an area and we get back the *z-score* that would have that area to the left in a standard normal curve.

The syntax is *invNorm(area)*

```
invNorm(.95)  
1.644853626
```

Another useful tool is *inverse normal* tool where we input an area and we get back the *z-score* that would have that area to the left in a standard normal curve.

The syntax is *invNorm(area)*

```
invNorm(.95)  
1.644853626
```

However, we often use the area to the right to denote this number.

$$z_{.05} = 1.644853626$$

Notice that $-z_{.025}$ and $z_{.025}$ give you more accurate bounds for where the middle 95% of the data lies in a normal distribution.

```
invNorm(.975)  
1.959963986
```

$$z_{.025} = 1.959963986$$

$$-z_{.025} = -1.959963986$$

The middle 95% of the data lies between $z=-1.96$ and $z=1.96$.

For most IQ tests, the scores are distributed normally with a *mean* of 100 and a *standard deviation* of 15. Find the following probabilities.

A person's IQ is between 90 and 110.

```
normalcdf(90,110  
,100,15)  
      .495015066
```

$$P(90 \leq x \leq 110) = .4950$$

For most IQ tests, the scores are distributed normally with a *mean* of 100 and a *standard deviation* of 15. Find the following probabilities.

A person's IQ is between 120 and 130.

```
normalcdf(120,130,100,15)
.0684612199
```

$$P(120 \leq x \leq 130) = .0685$$

For most IQ tests, the scores are distributed normally with a *mean* of 100 and a *standard deviation* of 15. Find the following probabilities.

A person's IQ is less than 70.

```
normalcdf(-99999  
9,70,100,15)  
.022750062
```

$$P(x < 70) = .0228$$

For most IQ tests, the scores are distributed normally with a *mean* of 100 and a *standard deviation* of 15. Find the following probabilities.

A person's IQ is greater than 140.

```
normalcdf(140,9999,100,15)  
.0038304251
```

$$P(x > 140) = .0038$$

For most IQ tests, the scores are distributed normally with a *mean* of 100 and a *standard deviation* of 15. Find the following probabilities.

A person's IQ is less than 152.

```
normalcdf(-99999  
9, 152, 100, 15)  
.9997364758
```

$$P(x < 152) = .9997$$

We can also find the cutoff score corresponding to the percentage or proportion that we want an IQ to be greater than. We just use the *invNorm* tool with the following syntax: *invNorm(proportion, mean, standard deviation)*.

What IQ is greater than that of 95% of the population?.

```
invNorm(.95,100,  
15)  
124.6728044
```

$IQ \approx 125$