

GRADIENTS TO LEVEL CURVES AND SURFACES - ANSWERS

For each function below, find parametric equations for the gradient vector at the point below on the indicated level curve or surface.

1. $z = x^2 + y^2$, level curve $z = 1$, $P = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$$\nabla z = 2x\hat{i} + 2y\hat{j}$$

$$\nabla z\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \sqrt{2}\hat{i} + \sqrt{2}\hat{j}$$

$$x = \frac{1}{\sqrt{2}} + t\sqrt{2}$$

$$y = \frac{1}{\sqrt{2}} + t\sqrt{2}$$

$$0 \leq t \leq 1$$

2. $z = x^2 - y^2$, level curve $z = 1$, $P = (1, 0)$.

$$\nabla z = 2x\hat{i} - 2y\hat{j}$$

$$\nabla z(1, 0) = 2\hat{i}$$

$$x = 1 + 2t$$

$$y = 0$$

$$0 \leq t \leq 1$$

3. $z = -x^2 - y^2$, level curve $z = -1$, $P = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$$\nabla z = -2x\hat{i} - 2y\hat{j}$$

$$\nabla z\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = -\sqrt{2}\hat{i} - \sqrt{2}\hat{j}$$

$$x = \frac{1}{\sqrt{2}} - t\sqrt{2}$$

$$y = \frac{1}{\sqrt{2}} - t\sqrt{2}$$

$$0 \leq t \leq 1$$

4. $z = x^2 + y^2 - 1$, level surface $x^2 + y^2 - z = 1$, $P = (1,1,1)$.

$$w = x^2 + y^2 - z$$

$$\nabla w = 2x\hat{i} + 2y\hat{j} - \hat{k}$$

$$\nabla w(1,1,1) = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$x = 1 + 2t$$

$$y = 1 + 2t$$

$$z = 1 - t$$

$$0 \leq t \leq 1$$

5. $z = x^2 - y^2 + 1$, level surface $x^2 - y^2 - z = -1$, $P = (1,1,1)$.

$$w = x^2 - y^2 - z$$

$$\nabla w = 2x\hat{i} - 2y\hat{j} - \hat{k}$$

$$\nabla w(1,1,1) = 2\hat{i} - 2\hat{j} - \hat{k}$$

$$x = 1 + 2t$$

$$y = 1 - 2t$$

$$z = 1 - t$$

$$0 \leq t \leq 1$$