JOINT DENSITY FUNCTIONS - ANSWERS

(1-3) Let $p(x,y)=\begin{cases}\frac{3}{2}x+3y & if\ \ 0\le x\le 1\ and\ \ 0\le y\le x\\ 0 & elsewhere\end{cases}$ be a joint density function.

1. Find the probability that $\frac{1}{2}\le x\le 1$ and $0\le y\le\frac{1}{2}$.

$$\int_{1/2}^{1}\int_{0}^{1/2}p(x,y)\,dydx=\int_{1/2}^{1}\int_{0}^{1/2}\left(\frac{3}{2}x+3y\right)dydx=\int_{1/2}^{1}\left(\frac{3}{2}xy+\frac{3y^2}{2}\right)\Bigg|_{0}^{1/2}dx$$

$$=\int_{1/2}^{1}\left(\frac{3x}{4}+\frac{3}{8}\right)dx=\left(\frac{3x^2}{8}+\frac{3x}{8}\right)\Bigg|_{1/2}^{1}=\left(\frac{3}{8}+\frac{3}{8}\right)-\left(\frac{3}{32}+\frac{3}{16}\right)$$

$$=\frac{15}{32}=0.46875$$

2. Find the probability that $\frac{1}{2}\le x\le 1$ and $0\le y\le x$.

$$\int_{1/2}^{1}\int_{0}^{x}p(x,y)\,dydx=\int_{1/2}^{1}\int_{0}^{x}\left(\frac{3}{2}x+3y\right)dydx=\int_{1/2}^{1}\left(\frac{3}{2}xy+\frac{3y^2}{2}\right)\Bigg|_{0}^{x}dx$$

$$=\int_{1/2}^{1}\left(\frac{3x^2}{2}+\frac{3x^2}{2}\right)dx=\int_{1/2}^{1}3x^2\,dx=x^3\Bigg|_{1/2}\ =1-\frac{1}{8}=\frac{7}{8}=0.875$$

3. Find the probability that $0\le y\le\frac{1}{2}$ and $y\le x\le\frac{1}{2}$.

$$\int_{0}^{1/2}\int_{y}^{1/2}p(x,y)\,dxdy=\int_{0}^{1/2}\int_{y}^{1/2}\left(\frac{3}{2}x+3y\right)dxdy=\int_{0}^{1/2}\left(\frac{3x^2}{4}+3xy\right)\Bigg|_{y}^{1/2}dy$$

$$=\int_{0}^{1/2}\left(\frac{3}{16}+\frac{3y}{2}\right)-\left(\frac{3y^2}{4}+3y^2\right)dy=\frac{3y}{16}+\frac{3y^2}{4}-\frac{y^3}{4}-y^3\Bigg|_{0}^{1/2}$$

$$=\frac{3}{32}+\frac{3}{16}-\frac{1}{32}-\frac{1}{8}=\frac{1}{8}=0.125$$

4. If $p(x)$ is a normal distribution with $\mu = 0$ and $\sigma = 1$ and if $q(y)$ is another normal distribution with $\mu = 0$ and $\sigma = 1$, then find the probability that $-1 \le x \le 1$ and $-1 \le y \le 1$. Set up a double integral and use **fnInt** on your TI-83/84 calculator to approximate numerically rounding to the nearest hundredth.

$$\int_{-1}^{1}\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}}\, dydx = \int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}\, dx \cdot \int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{\frac{-y^2}{2}}\, dy$$
$$\approx 0.6827 \cdot 0.6827 \approx 0.4661 \approx 0.47$$

5. If the weights of adult men are normally distributed with a mean of 200 pounds and a standard deviation of 10 pounds, and if IQ is normally distributed with a mean of 100 and a standard deviation of 15 points, then what is the probability that an adult male has a weight between 200 and 210 pounds and an IQ between 100 and 120? Let *x* equal weight and *y* equal IQ, set up a double integral, and use **fnInt** on your TI-83/84 calculator to approximate numerically rounding to the nearest hundredth.

$$\int_{200}^{210}\int_{100}^{120} \frac{1}{10\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-200}{10}\right)^2} \cdot \frac{1}{15\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-100}{15}\right)^2}\, dydx$$
$$= \int_{200}^{210} \frac{1}{10\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-200}{10}\right)^2} dx \cdot \int_{100}^{120} \frac{1}{15\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-100}{15}\right)^2} dy \approx 0.3413 \cdot 0.4088 \approx 0.1395 \approx 0.14$$