CONDITIONAL PROBABILITY

	democrat	republican	
male	20	30	50
female	40	10	50
	60	40	100

Suppose we have 100 people vote in an election, and the breakdown by party and gender is as indicated in the table below.

	democrat	republican	
male	20	30	50
female	40	10	50
	60	40	100

Often times we will want to ask question like, “What is the probability that someone is female given that they are democrat?”

	democrat	republican	
male	20	30	50
female	40	10	50
	60	40	100

We call this a *conditional probability*, and the condition redefines our sample space.

	democrat	republican	
male	20	30	50
female	40	10	50
	60	40	100

Below, we let *F* = *female* and *D* = *democrat.*

$$P(\text{female given democrat}) = P(F \mid D) = \frac{40}{60} = \frac{2}{3}$$

	democrat	republican	
male	20	30	50
female	40	10	50
	60	40	100

Notice that we can also write things as follows:

$$P(D) = 60/100$$

$$P(F\ \&\ D) = P(F \cap D) = 40/100$$

$$P(F \mid D) = \frac{40}{60} = \frac{40/100}{60/100} = \frac{P(F\ \&\ D)}{P(D)} = \frac{P(F \cap D)}{P(D)}$$

	democrat	republican	
male	20	30	50
female	40	10	50
	60	40	100

This leads to the following two formulas for *conditional probability.*

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \,\&\, B)}{P(B)}$$

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B \,\&\, A)}{P(A)} = \frac{P(A \,\&\, B)}{P(A)}$$

These formulas result in:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \& B)}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \& B)}{P(A)}$$

$$P(A \& B) = P(A \cap B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$$

Example: What is the probability of drawing two aces from a deck of 52 cards?

$$A = \text{ 1st card is an ace}$$

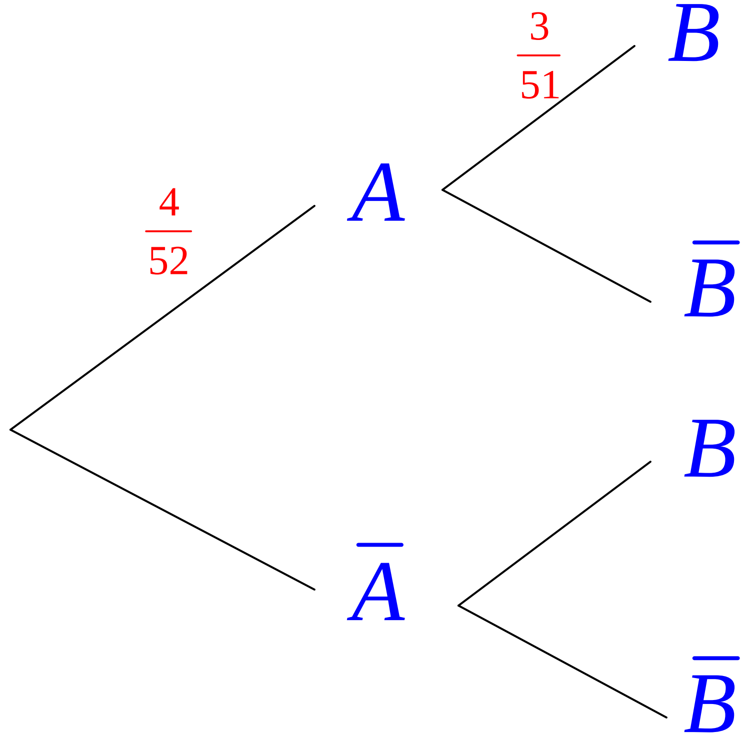
$$B = \text{2nd card is an ace}$$

$$P(A \,\&\, B) = P(A) \cdot P(B \mid A) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} \approx 0.0045$$

We could also illustrate this with a tree diagram.

$A =$ 1st card is an ace

$B =$ 2nd card is an ace



$$\frac{4}{52} \cdot \frac{3}{51} = P(A \,\&\, B)$$

Sometimes it doesn't matter whether *A* or *B* happens. The probability of the other event remains the same either way. When this happens, we say that the events are *independent.*

Independent events:

$$P(A) = P(A \mid B)$$

$$P(B) = P(B \mid A)$$

$$P(A \,\&\, B) = P(A \cap B) = P(A) \cdot P(B)$$

Experiment: If you flip a fair coin twice, what is the probability of getting two heads?

Experiment: If you flip a fair coin twice, what is the probability of getting two heads?

$A = \text{first flip is heads}$

$B = \text{2nd flip is heads}$

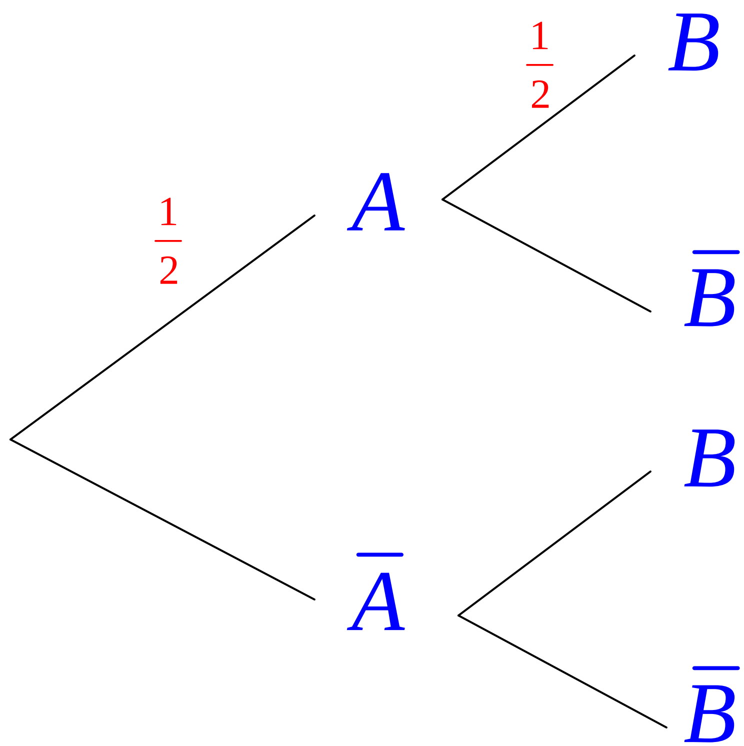
Clearly the events are independent

$$P(A \& B) = P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0.25$$

And here's a tree diagram!

$A = \text{first flip is heads}$

$B = \text{2nd flip is heads}$



$$\frac{1}{2} \cdot \frac{1}{2} = P(A \,\&\, B)$$

Question: Do *independence* and *mutually exclusive* mean the same thing?

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No, they are totally opposite!