

# COUNTING



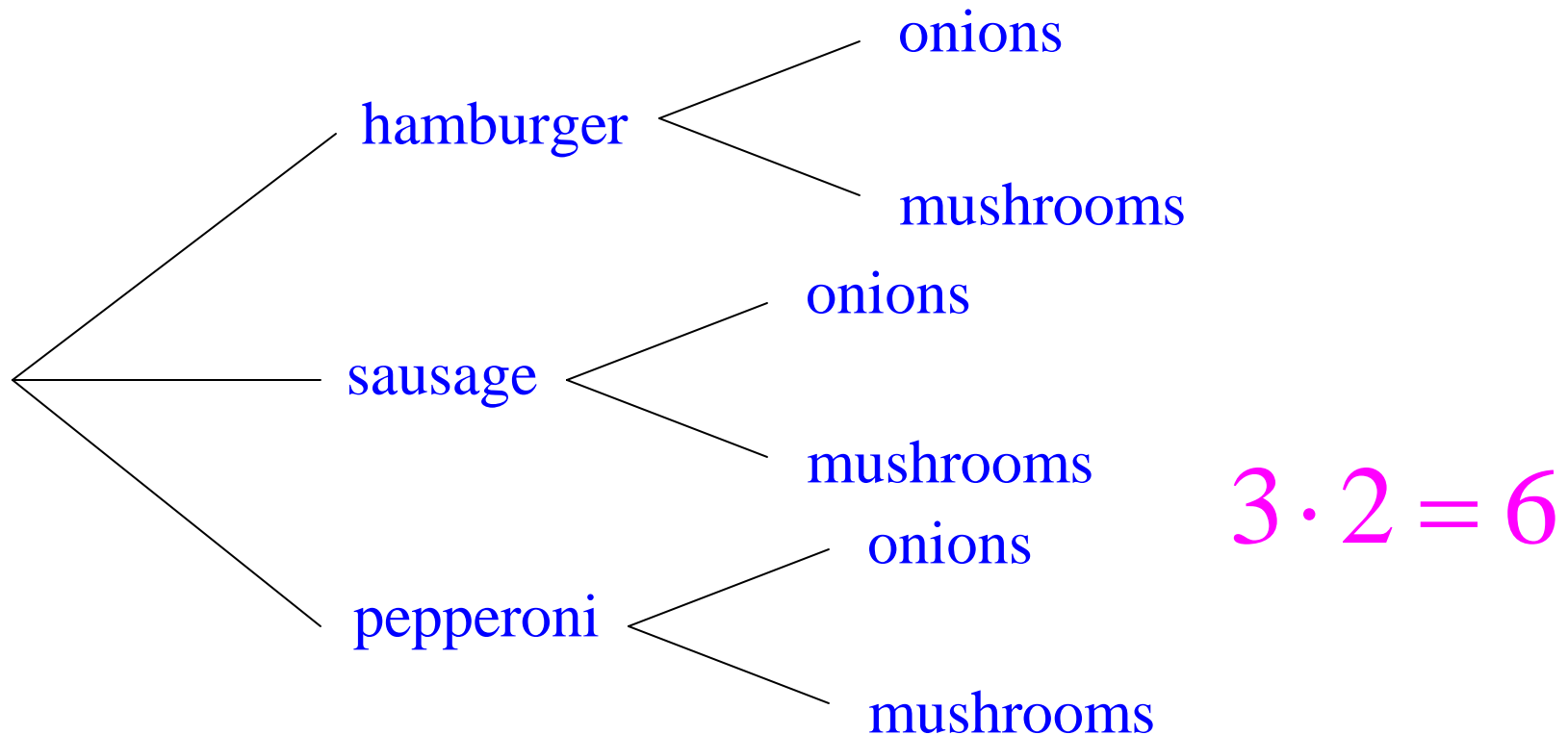
## FUNDAMENTAL COUNTING RULE:

For a sequence of two events, if the first event can happen in  $m$  ways and the second event can happen in  $n$  ways, then together the events can happen in  $m \times n$  ways.

## Example:

You select one meat topping and one veggie topping for a pizza. For the meats you can get hamburger, sausage, or pepperoni, and for the veggies you can get onions or mushrooms. How many possibilities are there?

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Should a *combination lock* be called a *permutation lock*?

How many permutations can we make of the letters in the set  $A$ ? (draw without replacement)

$$A = \{a, b, c\}$$

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$$3 \cdot 2 \cdot 1 = 6$$

*abc bac cab*

*acb bca cba*

How many combinations are represented below?

$$A = \{a, b, c\}$$

*abc bac cab*

*acb bca cba*

How many combinations are representd below?

$$A = \{a, b, c\}$$

Only one!

*abc bac cab*

*acb bca cba*

Definition: The product  $(n)(n-1)(n-2) \dots (1)$  is called *n factorial* and denoted by *n!*.

By definition,  $0! = 1$ .

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$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$\frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 5 \cdot 4 = 20$$



Below is a formula for counting the number of permutations of  $n$  objects if we choose only  $r$ .

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$${}_5 P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 5 \cdot 4 = 20$$

Now we look at the formula for counting the number of combinations of  $n$  objects if we choose only  $r$ .

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$${}_n C_r = \frac{n!}{(n-r)!r!}$$

$${}_5 C_2 = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} = \frac{120}{6 \cdot 2} = 10$$

How many different committees of 5 can we form from 20 people?

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$${}_{20}C_5 = \frac{20!}{15!5!} = 15,504$$

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$${}_{15}P_5 = \frac{15!}{10!} = 360,360$$



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$$5 \cdot 10 = 50$$

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$$\binom{5}{2} \binom{10}{2} = 10 \cdot 45 = 450$$

How many different five-card poker hands are possible?

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$${}_{52}C_5 = 2,598,960$$

How many different five-card hands are possible if we draw  
*with replacement?*  
*(and count permutations instead of combinations)*

$$52 \cdot 52 \cdot 52 \cdot 52 \cdot 52 = 52^5 = 380,204,032$$

How many different permutations are possible of the letters in the word *meat*?



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$$4! = 24$$

How many different permutations are possible of the letters in the word *meef*?

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$$\frac{4!}{2!} = \frac{24}{2} = 12$$