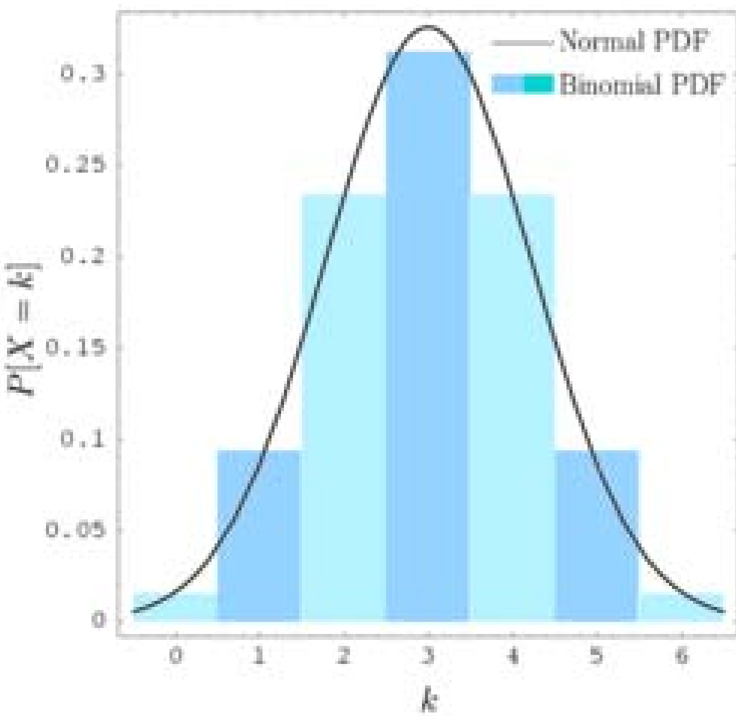
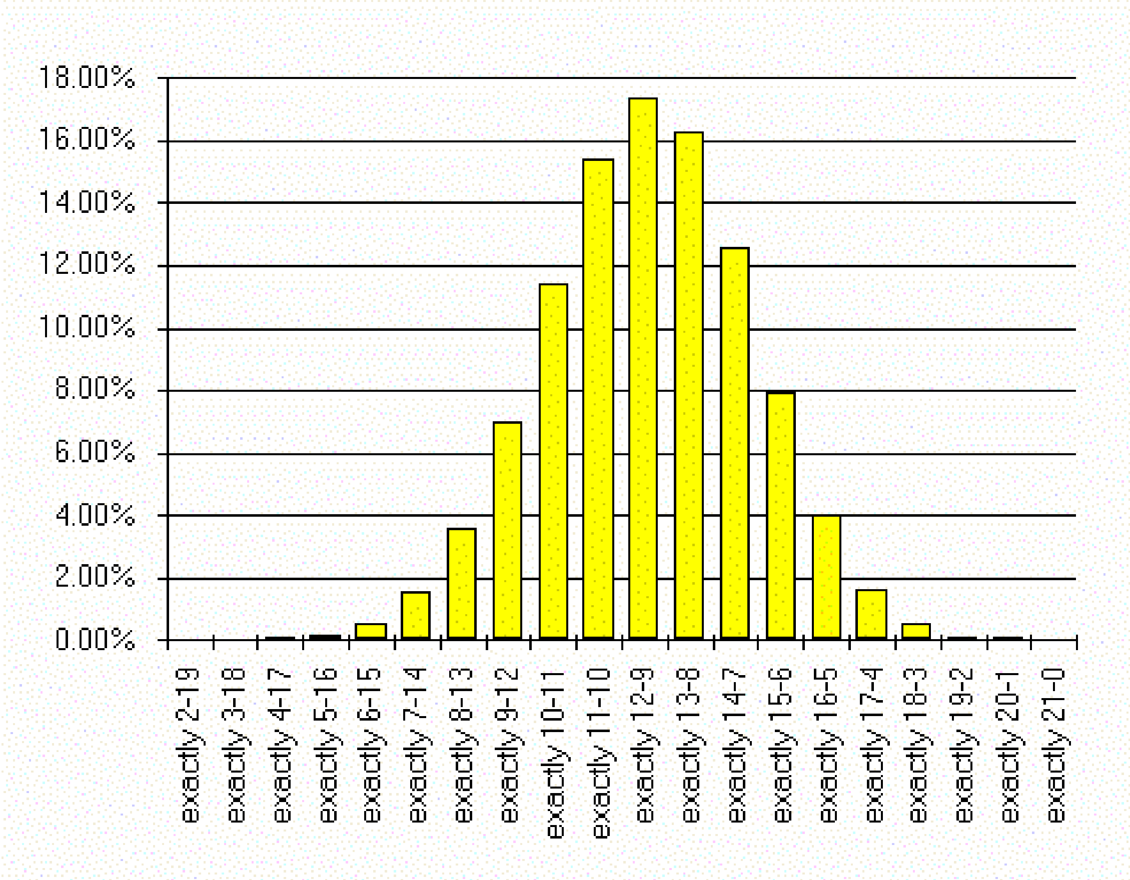
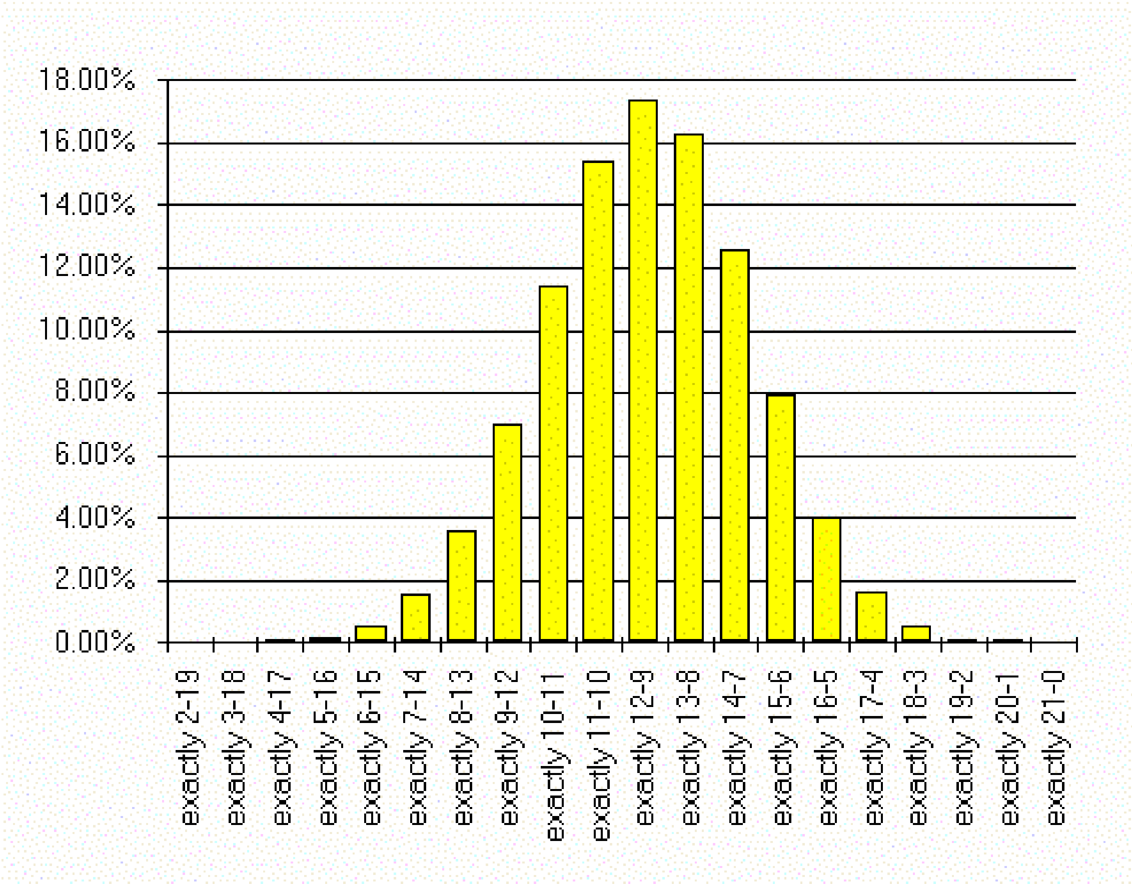
THE NORMAL APPROXIMATION OF THE BINOMIAL DISTRIBUTION



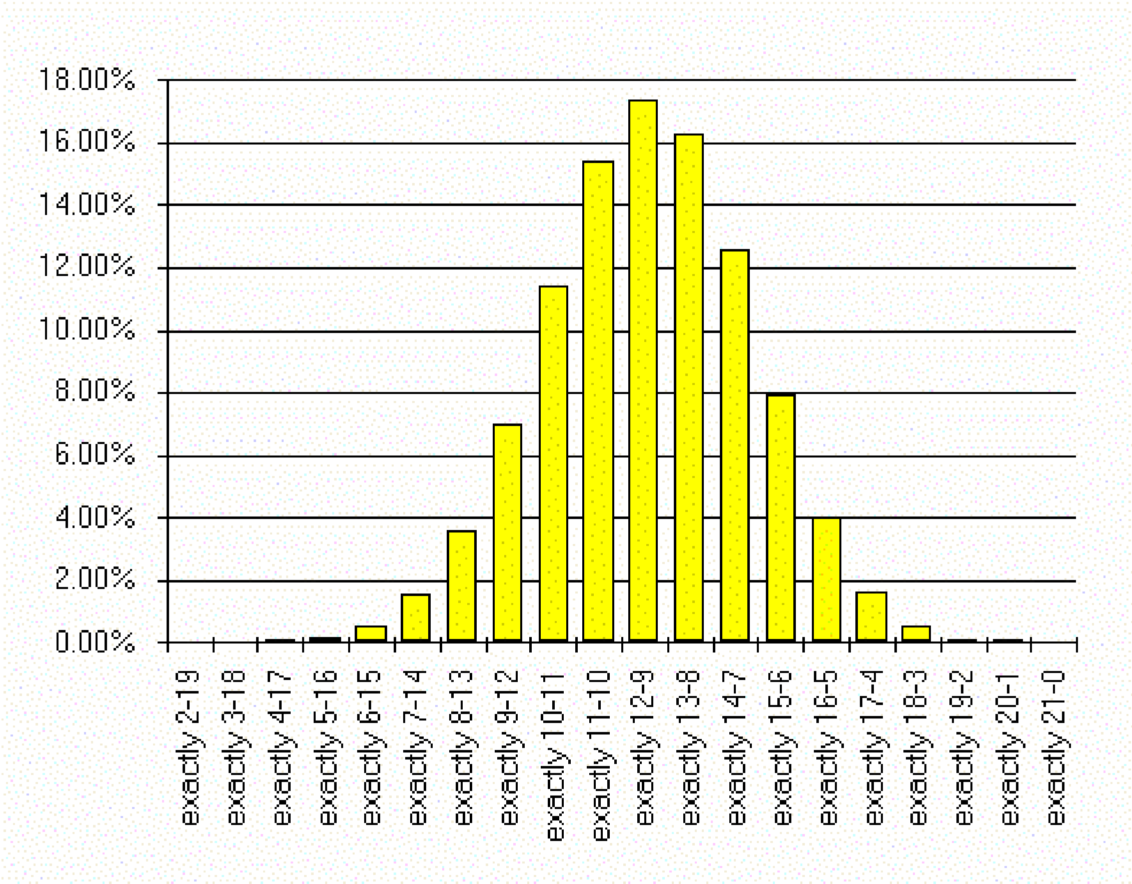
Quite often, a binomial distribution will sort of like a discrete version of a normal distribution.



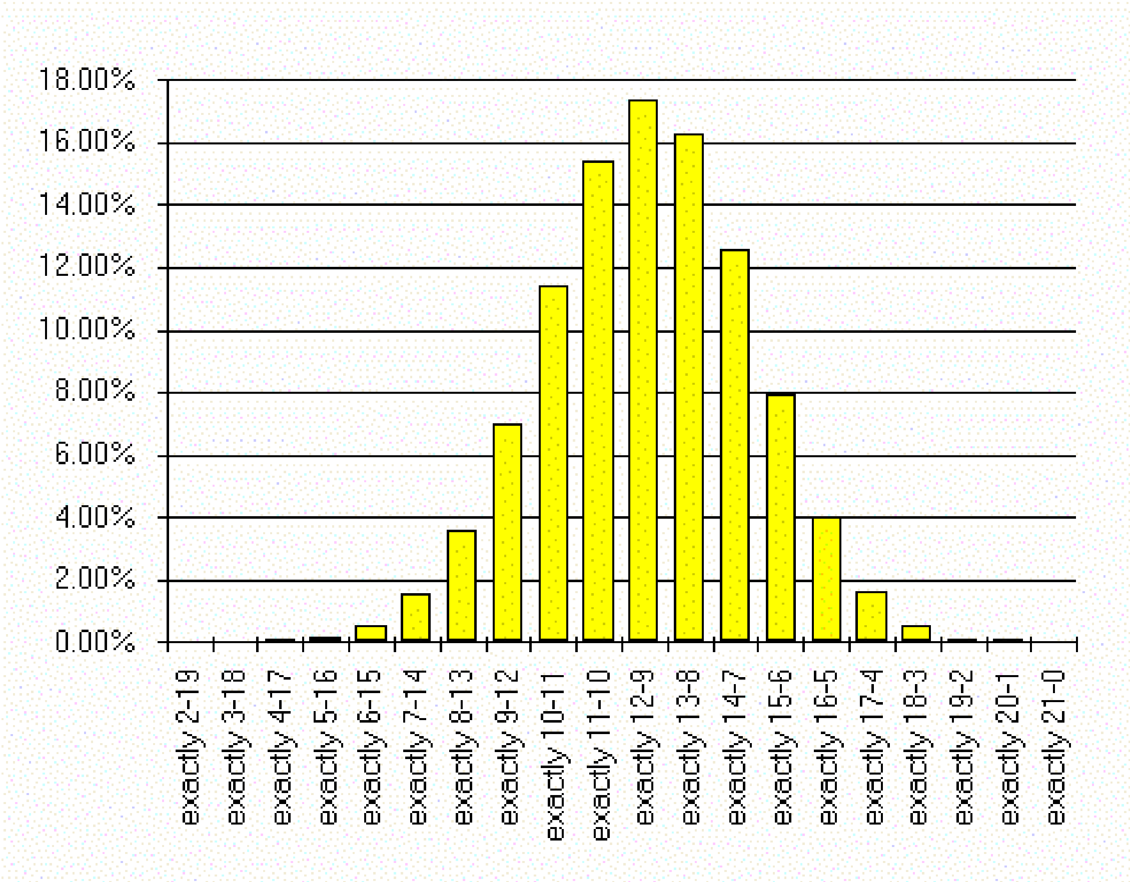
When this happens, we can use a corresponding normal distribution to estimate probabilities in a binomial distribution.



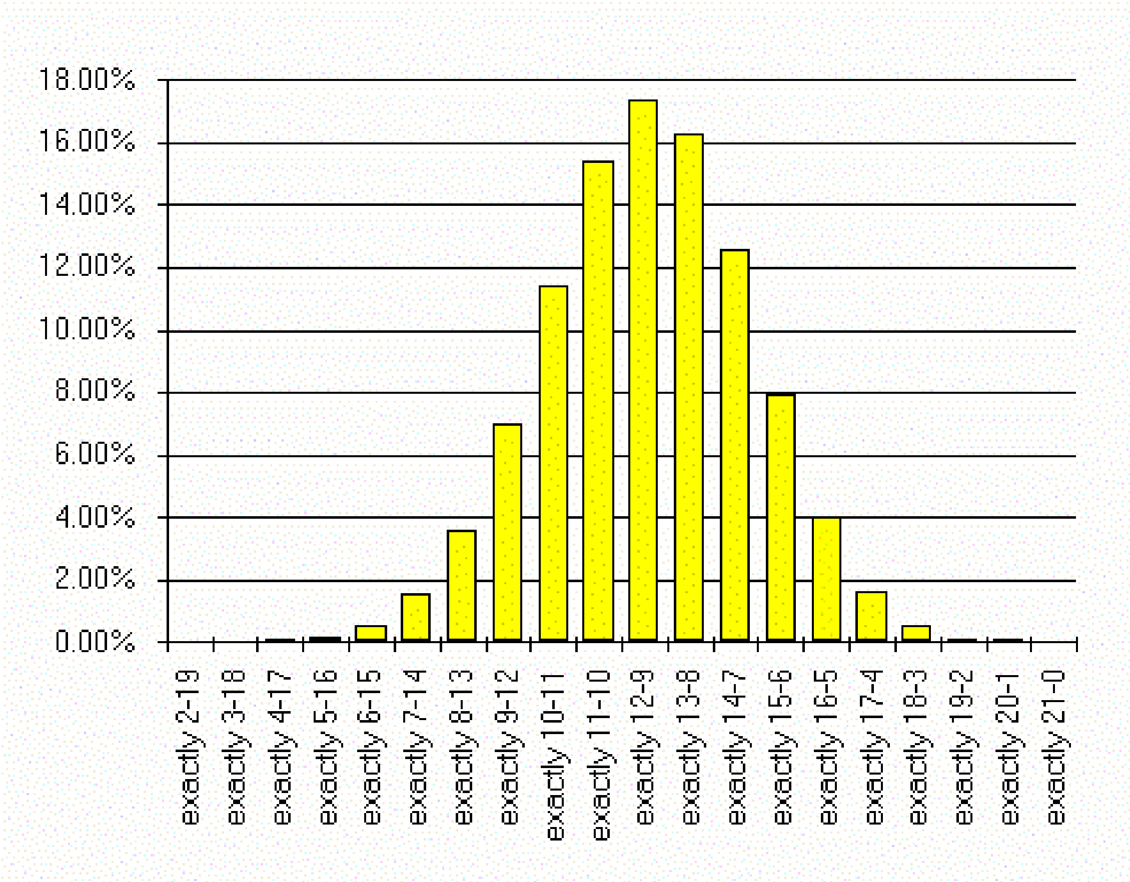
Our criteria for doing this will be that both $np \geq 5$ and $nq \geq 5$.



Suppose we flip a fair coin 100 times, and X = *number of heads*.



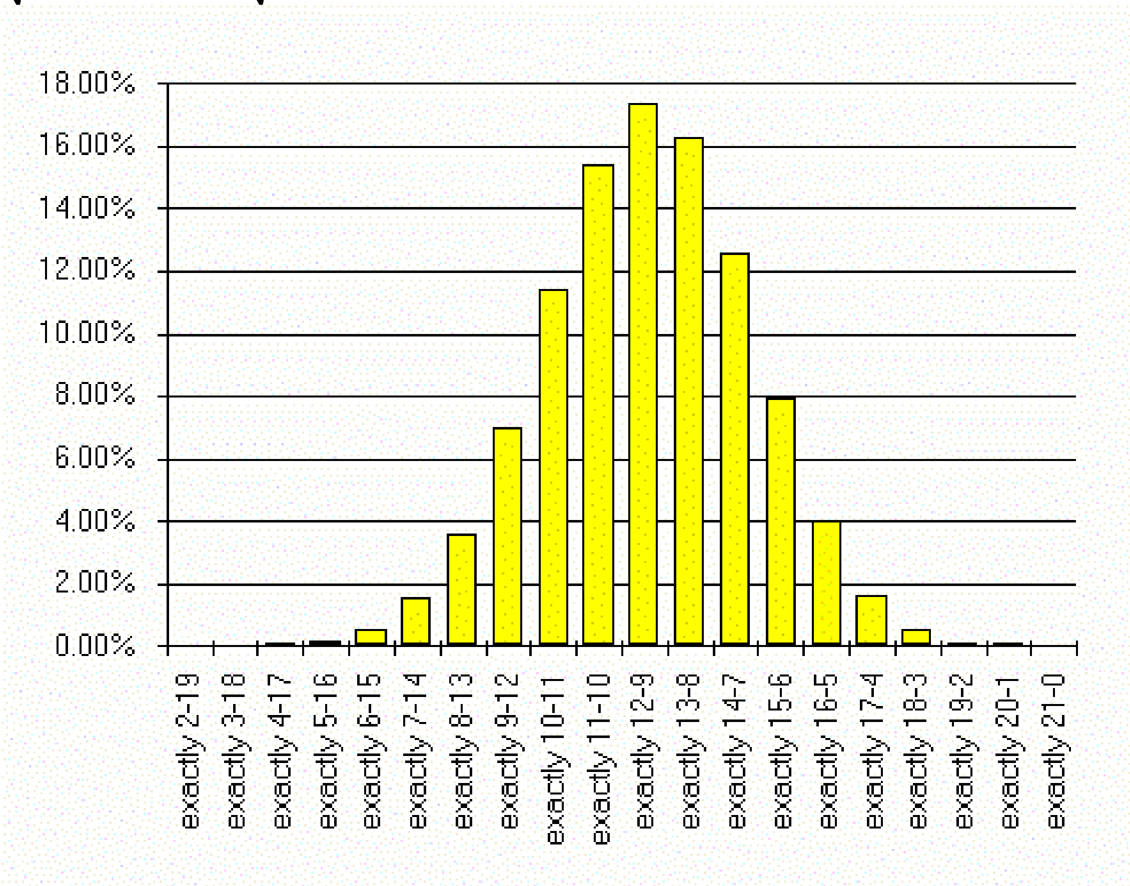
Then since *np* = *(100)(.5) = 50,* and *nq* = *(100)(.5)=50,* we can use a corresponding normal distribution to approximate binomial probabilities.



The mean of our normal distribution will be

$\mu = np = (100)(.5) = 50$, and the standard deviation will be

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5.$$



Now let's estimate the probability that *x = 60.* There's one problem we have to take care of first, though. Since the binomial distribution is discrete, but the normal distribution is continuous, we have to make what we call the *correction for continuity.*

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

What this means is that we now have to think of the value 60 as extending across a continuum from 59.5 to 60.5. Given that correction, we can now easily estimate the probability using our calculator.

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$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

$$P(x = 60) \approx P(59.5 \leq x \leq 60.5)$$

$$= \text{normalcdf}\,(59.5, 60.5, 50, 5) \approx 0.0109$$

And this is not far off from what we would get if we tried to compute the binomial probability directly.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

$$P(x = 60) \approx P(59.5 \leq x \leq 60.5)$$

$$= \text{normalcdf}\,(59.5, 60.5, 50, 5) \approx 0.0109$$

$$P(x = 60) = {}_{100}C_{60}\,(.5^{60})(.5^{40})$$

$$= \text{binompdf}\,(100, .5, 60) = 0.0108$$

Now let's find the probability that *x* is less than 60, and remember to make the correction for continuity.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

Now let's find the probability that *x* is less than 60, and remember to make the correction for continuity.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

$$P(x < 60) \approx P(x < 59.5)$$

$$= \text{normalcdf}(-999999, 59.5, 50, 5) \approx 0.9713$$

Contrast this last result with the probability that *x* is at most 60.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

Contrast this last result with the probability that *x* is at most 60.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

$$P(x \leq 60) \approx P(x \leq 60.5)$$
$$= \text{normalcdf}(-999999, 60.5, 50, 5) \approx 0.9821$$

Next, let’s look at the probability that *x* is more than 60.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

Next, let's look at the probability that *x* is more than 60.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

$$P(x > 60) \approx P(x > 60.5)$$

$$= \text{normalcdf}(60.5, 999999, 50, 5) \approx 0.0179$$

At finally, let's look at the probability that *x* is at least 60.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

At finally, let's look at the probability that *x* is at least 60.

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

$$P(x \geq 60) \approx P(x \geq 59.5)$$

$$= \text{normalcdf}\,(59.5, 999999, 50, 5) \approx 0.0287$$

At finally, let's look at the probability that *x* is at least 60.

And that's all there is to it!

$$\mu = np = (100)(.5) = 50$$

$$\sigma = \sqrt{npq} = \sqrt{(100)(.5)(.5)} = \sqrt{25} = 5$$

$$P(x \geq 60) \approx P(x \geq 59.5)$$

$$= \text{normalcdf}(59.5, 999999, 50, 5) \approx 0.0287$$