ANTIDERIVATIVES

$$\int f(x)\,dx = F(x) + c$$

Often times we begin with a function that we know is a derivative of another function, and we want to find that second function.

We call that second function an antiderivative of the first function.

Also, for convenience, we'll often denote the derivative by *f(x)* and the antiderivative by *F(x)*.

For example, think about what might be the antiderivative of the function below.

$$f(x) = 3x^2$$

When we differentiate *x* to a power, we multiply by the exponent and then decrease the exponent by 1.

$$f(x) = 3x^2$$

To get the antiderivative, we just reverse our steps. We add 1 to the exponent and then divide.

$$f(x) = 3x^2$$

$$F(x) = \frac{3x^{2+1}}{2+1} = \frac{3x^3}{3} = x^3$$

However, this is not the only antiderivative of *f(x)*. If we add any constant term to *F(x)*, we get another antiderivative.

$$f(x) = 3x^2$$

$$F(x) = \frac{3x^{2+1}}{2+1} = \frac{3x^3}{3} = x^3$$

$$F_2(x) = x^3 + 5$$

In general, the family or collection of all antiderivatives of *f(x)* is given by any particular antiderivative *F(x)* plus an arbitrary constant.

$$f(x) = 3x^2$$

$$F(x) = x^3 + c = \text{ family of all antiderivatives of } f(x)$$

For reasons which won't be made clear until later on, we call the family of all antiderivative of *f(x)* the *indefinite integral of f(x),* and we denote this by an elongated *s* that we call the *integral sign.*

$$f(x) = 3x^2$$

$$\int 3x^2\,dx = x^3 + c = \text{ family of all antiderivatives of } f(x)$$

integral sign

constant of integration

variable we are integrating with respect to

Theorem: If $f(x) = x^n$, $n \neq -1$, then $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c.$

Example:

$$\int x^4 \, dx = ?$$

Example:

$$\int x^4 \, dx = ?$$

$$\int x^4 \, dx = \frac{x^5}{5} + c$$

Example:

$$\int x^{10}\,dx = ?$$

Example:

$$\int x^{10}\,dx = ?$$

$$\int x^{10}\,dx = \frac{x^{11}}{11} + c$$

Example:

$$\int x^{1/2}\,dx = ?$$

Example:

$$\int x^{1/2}\,dx = ?$$

$$\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} + c = \frac{2x^{3/2}}{3} + c$$

Example:

$$\int x^{-3}\,dx = ?$$

Example:

$$\int x^{-3}\,dx = ?$$

$$\int x^{-3}\,dx = \frac{x^{-2}}{-2} + c = -\frac{1}{2x^{2}} + c$$

Since constant factors are not affected by the differentiation process, they are also not effected by the integration process.

$$\int 5x^2\,dx = \frac{5x^3}{3} + c$$

Also, since we can differentiate a function term by term, we can likewise integrate a function term by term.

$$\int (5x^2 + 2x + 3)\,dx = \frac{5x^3}{3} + x^2 + 3x + c$$

Since the derivative of $f(x)=e^x$ is e^x, an antiderivative of e^x is also e^x.

$$\int e^x \, dx = e^x + c$$

An antiderivative of *f(x)=b^x* is *b^x/ln(b)*. We can verify this by differentiating.

Since $\frac{d\frac{b^x}{\ln b}}{dx} = \frac{1}{\ln b} b^x \ln b = b^x$, it follows that

$$\int b^x \, dx = \frac{b^x}{\ln b} + c.$$

An antiderivative of ***x^{-1} = 1/x*** **is** ***ln|x|.*** **Again, this is true since the derivative of** ***ln|x|*** **is** ***1/x.***

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + c$$

SUMMARY

1. $\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

2. $\int x^{-1} \, dx = \ln|x| + c$

3. $\int e^x \, dx = e^x + c$

4. $\int b^x \, dx = \dfrac{b^x}{\ln b} + c$

5. $\int kf(x) \, dx = k \int f(x) \, dx \quad (k \text{ a constant})$

6. $\int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx$

Example:

$$\int\left[5x^3+10e^x+2^x+\frac{4}{x}\right]dx=?$$

Example:

$$\int \left[5x^3 + 10e^x + 2^x + \frac{4}{x} \right] dx = ?$$

$$\int \left[5x^3 + 10e^x + 2^x + \frac{4}{x} \right] dx = \frac{5x^4}{4} + 10e^x + \frac{2^x}{\ln 2} + 4\ln|x| + c$$

If we are given an *initial condition*, then we can generally find a specific value for our constant of integration.

$f(x) = 2x + 5$

$F(x)$ is an antiderivative of $f(x)$

$F(0) = 3$

Find $F(x)$.

If we are given an *initial condition*, then we can generally find a specific value for our constant of integration.

$$f(x) = 2x + 5$$

$F(x)$ is an antiderivative of $f(x)$

$$F(0) = 3$$

Find $F(x)$.

$$F(x) = \int (2x+5)\,dx = x^2 + 5x + c$$

$$F(0) = 3 = 0^2 + 5(0) + c = c \Rightarrow F(x) = x^2 + 5x + 3$$

The marginal cost to produce baseball caps at a production level of *x* caps is *4 – 0.001x* dollars per cap, and the cost of producing 100 caps is $500. Find the cost function.

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$$C(x) = \int (4 - 0.001x)\, dx = 4x - 0.001\frac{x^2}{2} + c$$

$$= 4x - 0.0005x^2 + c$$

The marginal cost to produce baseball caps at a production level of *x* caps is *4 – 0.001x* dollars per cap, and the cost of producing 100 caps is $500. Find the cost function.

$$C(x) = \int (4 - 0.001x)\,dx = 4x - 0.001\frac{x^2}{2} + c$$

$$= 4x - 0.0005x^2 + c$$

$$C(100) = 500 = 4(100) - 0.0005(100^2) + c$$

$$= 395 + c \Rightarrow c = 105$$

$$\Rightarrow C(x) = 4x - 0.0005x^2 + 105 \text{ dollars}$$

The velocity of a particle moving along a straight line is given by $v(t) = 4t + 1$ *m/s.* Given that the particle is at position $s = 2$ at time $t = 1$, find the position function $s(t)$.

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$$= 2t^2 + t + c$$

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$$= 2t^2 + t + c$$

$$s(1) = 2 = 2(1^2) + 1 + c$$

$$= 3 + c \Rightarrow c = -1$$

$$\Rightarrow s(t) = 2t^2 + t - 1 \text{ meters}$$