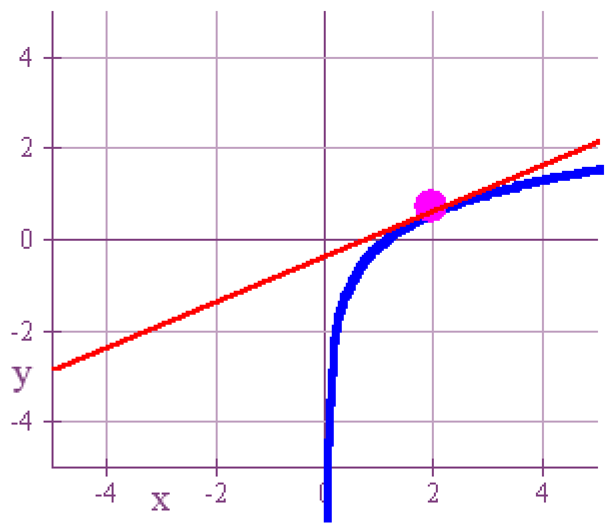
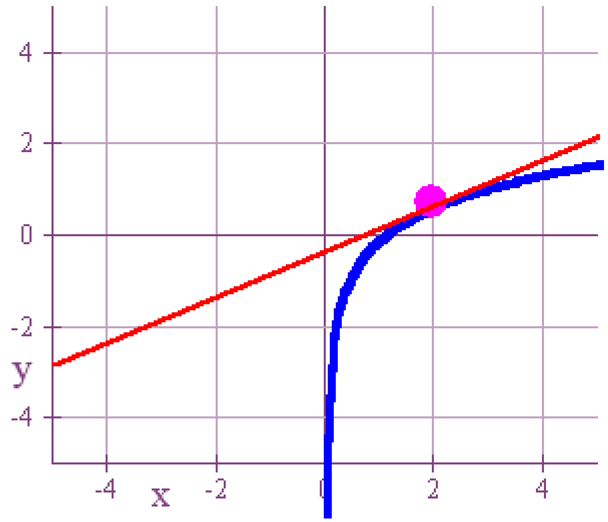
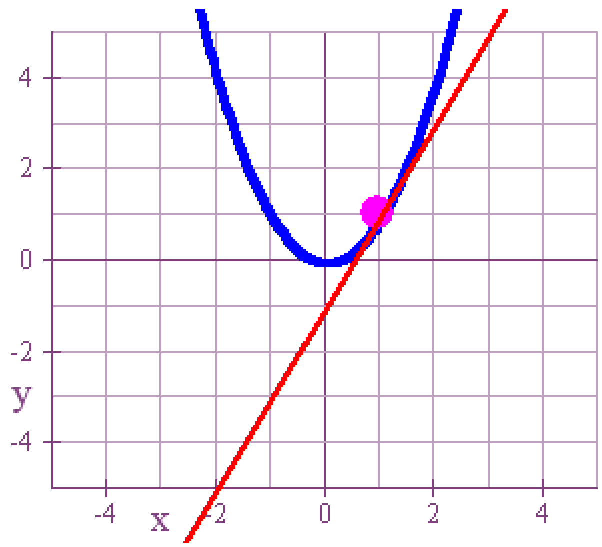
Finding Derivatives Graphically



Recall that the derivative of a function at a point is also the slope of the tangent line at that point. Thus, we can often figure out things about a derivative just by studying the graph of a function and its tangent lines.



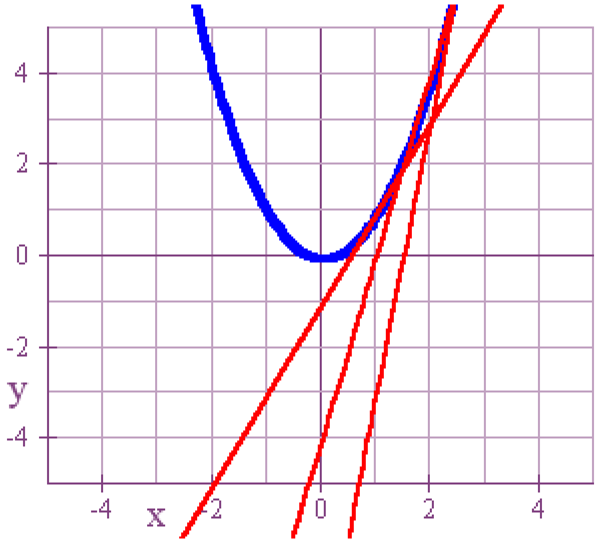
Below is the graph of *$f(x)=x^2$* with the tangent line drawn in at *$x=1$*. In this case, we can use the grid along with the formula *slope=rise/run* to figure out that the slope of the tangent line is 2.



$$f(x) = x^2$$

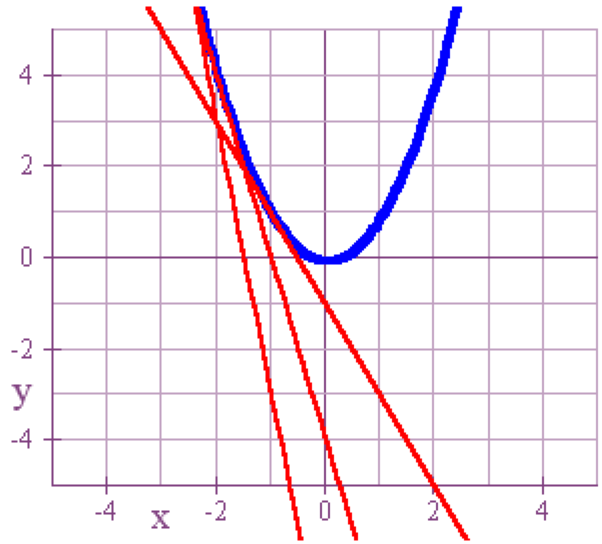
$$f'(1) = 2$$

Notice also that our function is increasing when *x>0*, and that the tangent lines for *x>0* all have positive slope.



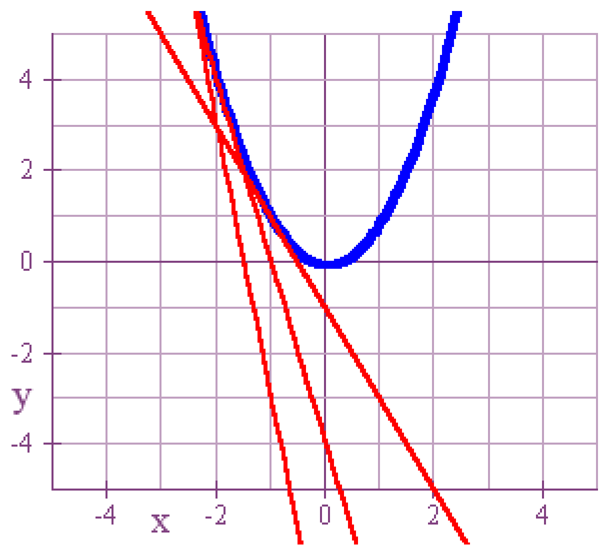
$$f(x) = x^2$$

**Similarly, our function is decreasing when *x<0*,
and the tangent lines for *x<0* all have negative slope.**



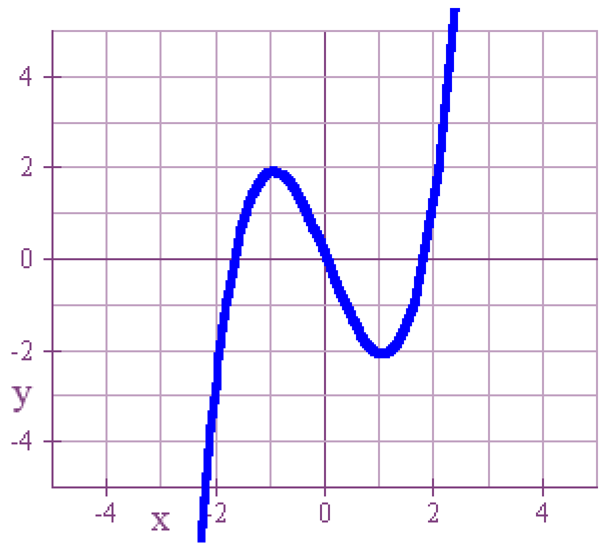
$$f(x) = x^2$$

This suggests that derivatives are positive when a function is increasing, and derivatives are negative when a function is decreasing. This will become more important to us later on.



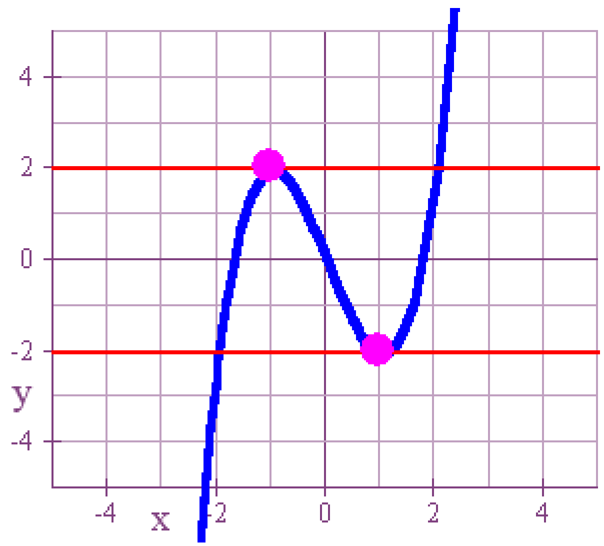
$$f(x) = x^2$$

Now let's look at *f(x)=x³-3x*. This graph has a peak at *(*-1,2) and a valley at *(1,-2)*.



$$f(x) = x^3 - 3x$$

The graph also has a horizontal tangent line at each of these points. Thus, the derivatives are zero at these points.

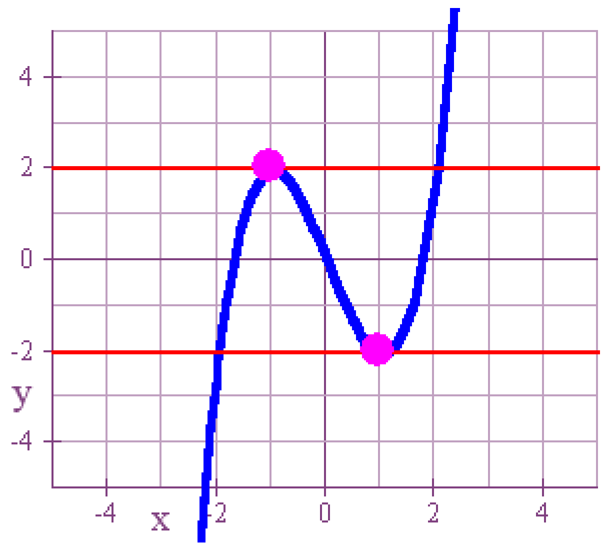


$$f(x) = x^3 - 3x$$

$$f'(-1) = 0$$

$$f'(1) = 0$$

The graph also has a horizontal tangent line at each of these points. Thus, the derivatives are zero at those points. This will also become important to us later on.

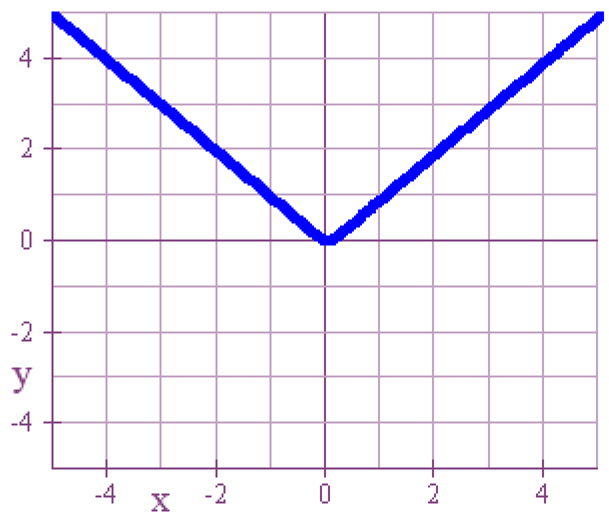


$$f(x) = x^3 - 3x$$

$$f'(-1) = 0$$

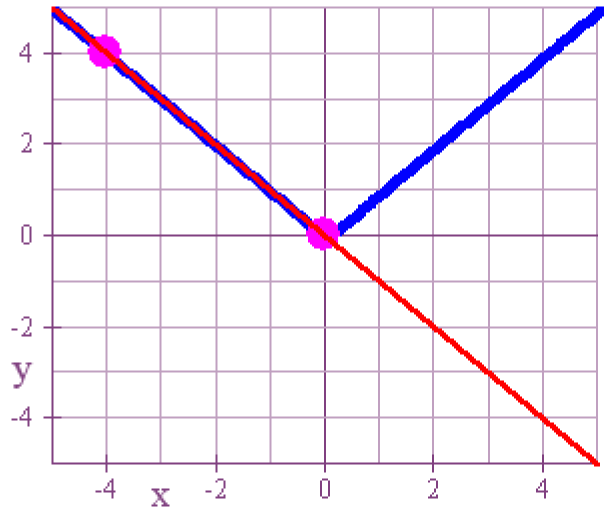
$$f'(1) = 0$$

Here's something a little different, the graph of ***f(x)=|x|.***



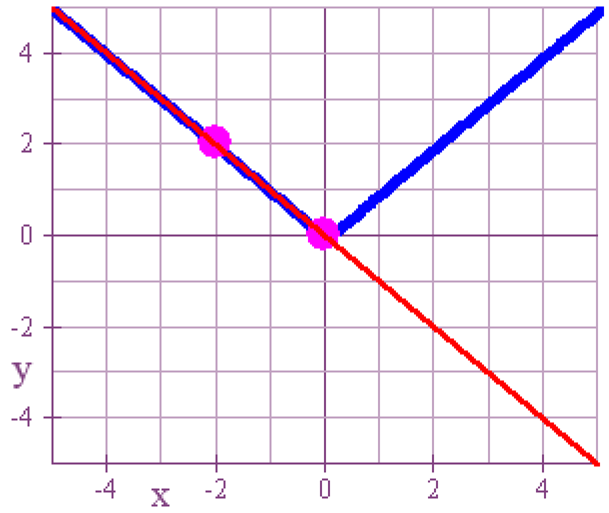
$$f(x) = |x|$$

If we look at the secant line through *(0,0)* and a second point to the left of *(0,0)*, then this secant line has negative slope.



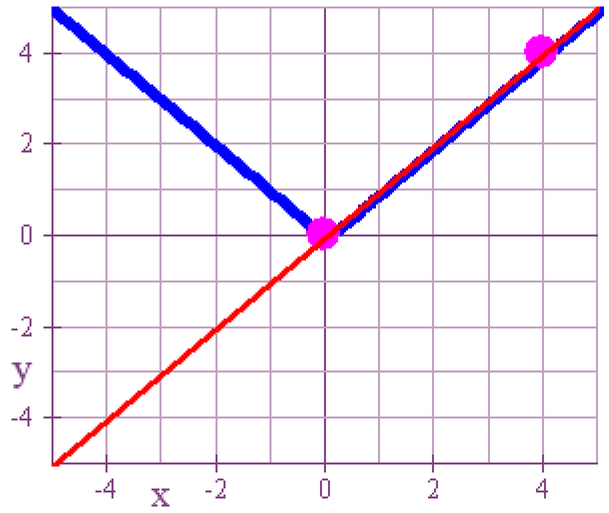
$$f(x) = |x|$$

Furthermore, moving our second point closer to the origin doesn't change the behavior of our secant line.



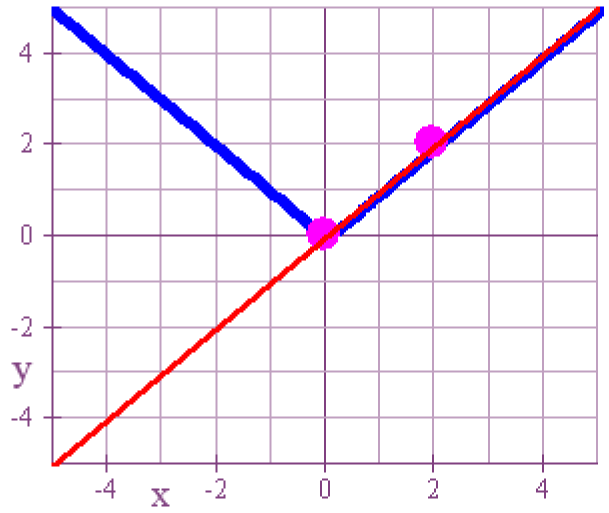
$$f(x) = |x|$$

On the other hand, if our second point is to the right of the origin, then the secant line has positive slope.



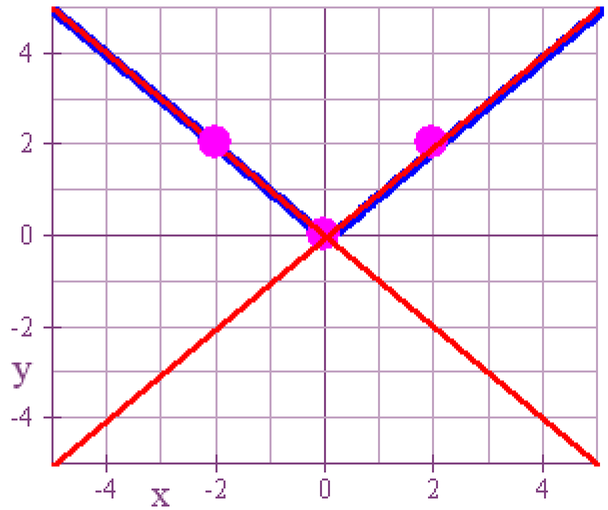
$$f(x) = |x|$$

And again, moving this second point a little closer to the origin doesn't change the behavior of the secant line.



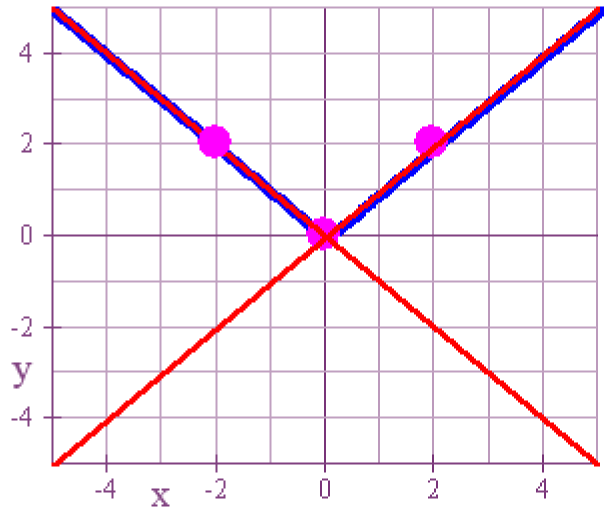
$$f(x) = |x|$$

The bottom line is that we can't define either a tangent line or a derivative at *(0,0).*



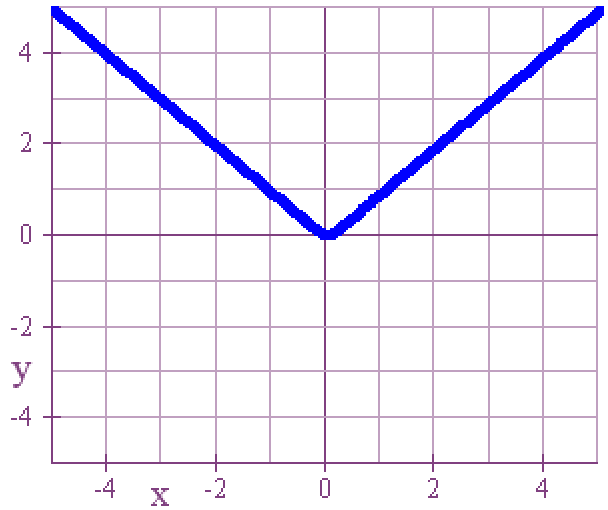
$$f(x) = |x|$$

This is because secant lines from one direction always have a fixed negative slope while those coming from the other direction have a fixed positive slope. They never approach a common tangent line.



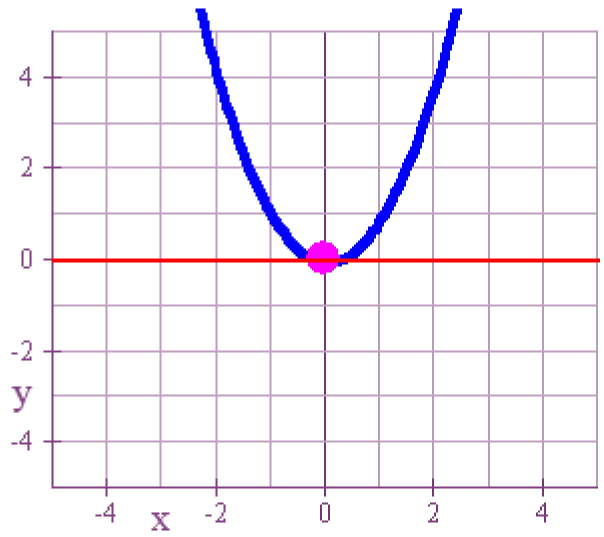
$$f(x) = |x|$$

In general, wherever you have a sharp, corner point on a graph, the derivative and tangent line are undefined.



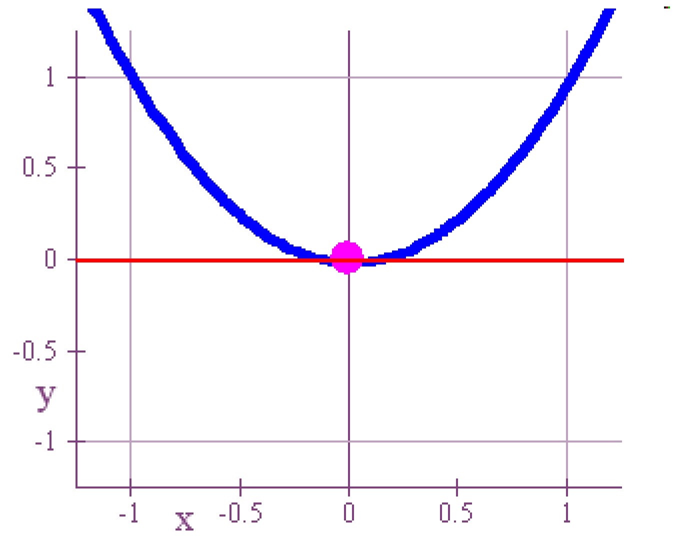
$$f(x) = |x|$$

Now here's another interesting fact. Suppose we examine the graph of *$f(x)=x^2$* along with its tangent line at the origin.



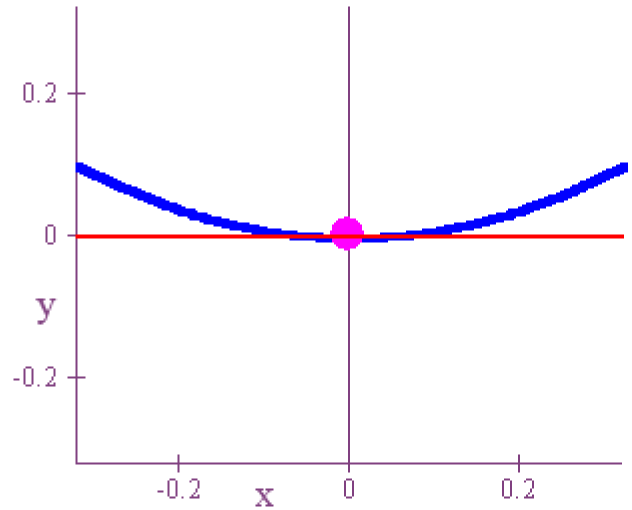
$$f(x) = x^2$$

Watch what happens as we zoom in at the origin.



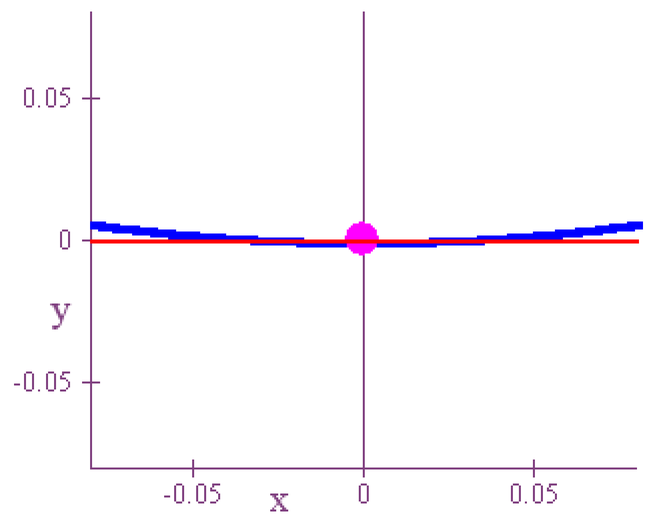
$$f(x) = x^2$$

And again



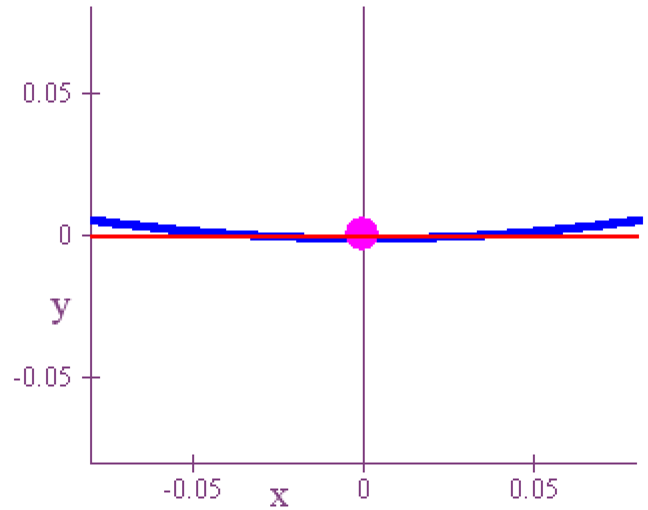
$$f(x) = x^2$$

And again



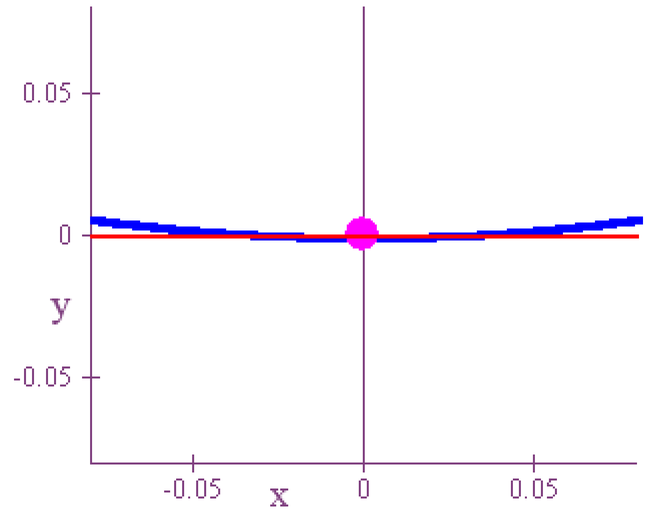
$$f(x) = x^2$$

The more we zoom in, the flatter that tiny piece of the curve becomes, and the more it resembles its own tangent line at that point.



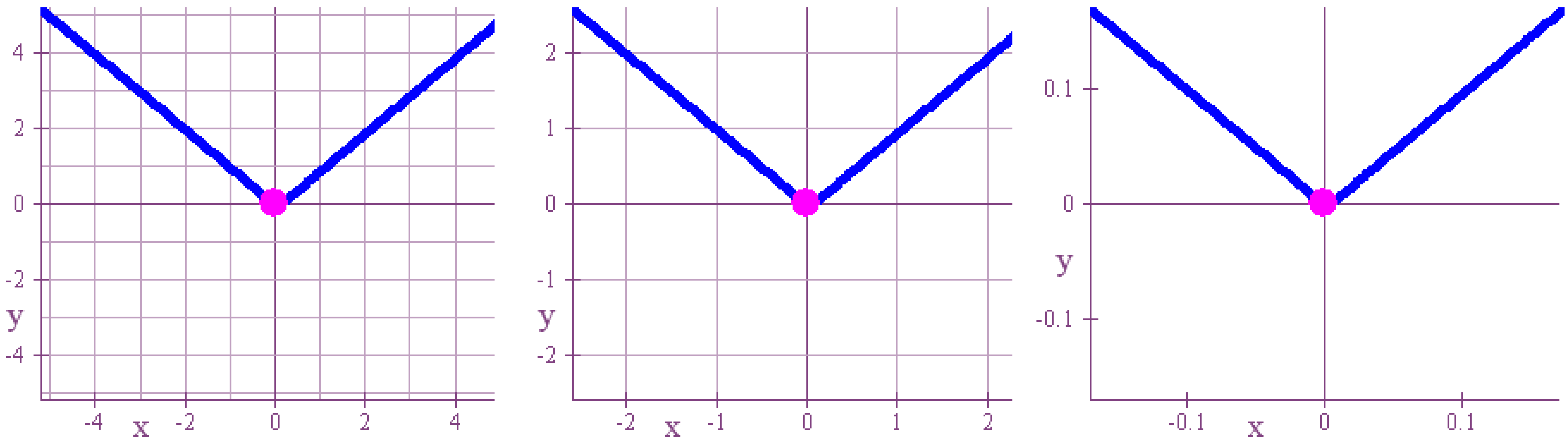
$$f(x) = x^2$$

We call this property *local linearity*, and a tangent line exists at a point only if the graph is *locally linear* at that point.



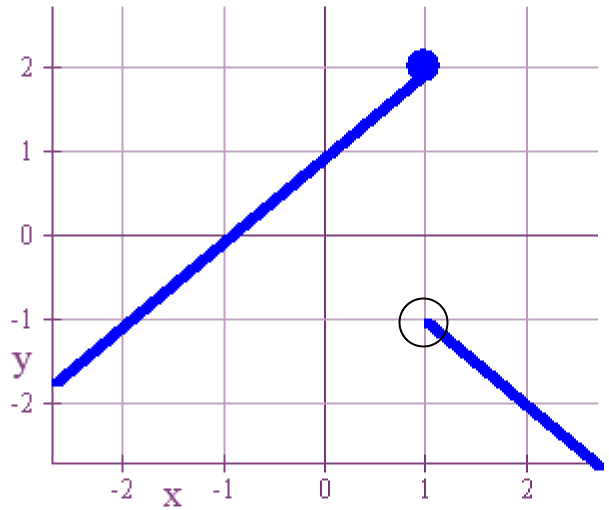
$$f(x) = x^2$$

Notice that when you have a sharp point on a graph, the graph is not *locally linear* at that point, and hence, doesn't have a tangent line there.



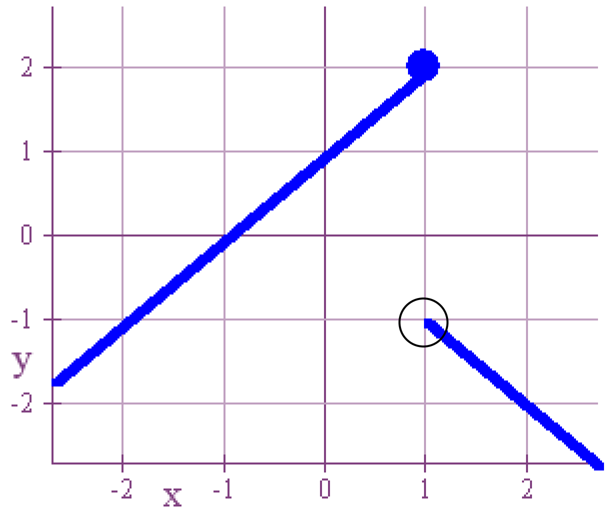
$$f(x) = |x|$$

And as a final note, there is a theorem in calculus that says that if a function has a derivative at a point, then it must be continuous at that point.



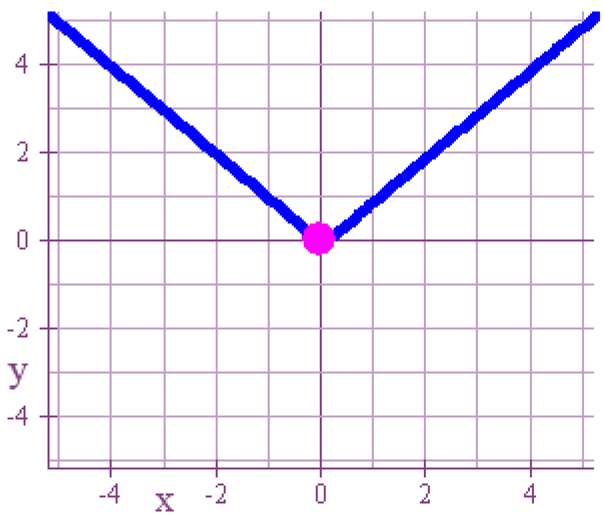
$$f(x)=\begin{cases} x+1 & x\le 1 \\ -x & x>1 \end{cases}$$

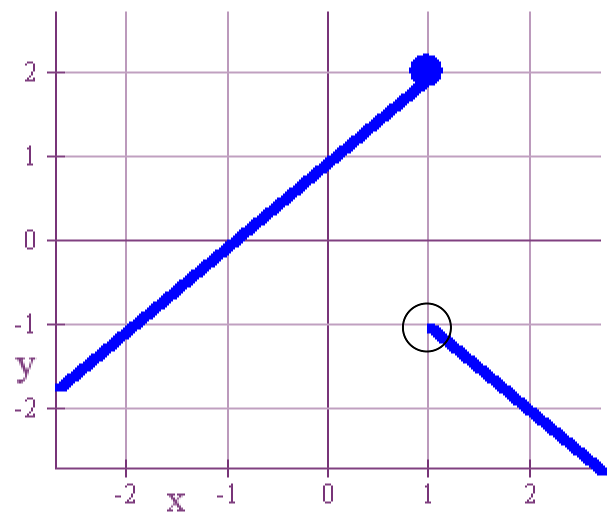
Hence, the function below has neither a tangent line nor a derivative at *x=1*, since it is not continuous at *x=1*.



$$f(x)=\begin{cases} x+1 & x\le 1 \\ -x & x>1 \end{cases}$$

The bottom line is that derivatives fail to exist at both discontinuities and at sharp, corner points.





$$f(x) = |x|$$

$$f(x) = \begin{cases} x + 1 & x \leq 1 \\ -x & x > 1 \end{cases}$$