Finding Derivatives Numerically

Recall how we defined *average rate of change* over an interval *(a,b)* for a function *y=f(x).*

$$\text{Average Rate of Change } = \frac{f(b) - f(a)}{b-a}$$

Also, if we let our second point be variable and denote it by *(x,f(x)),* then we can rewrite our *average rate of change formula* as follows.

$$\text{Average Rate of Change } = \frac{f(x) - f(a)}{x - a}$$

We now want to transition from *average rate of change* to *instantaneous rate of change*. And how do we do this? Simple! We just move our second point *(x,f(x))* closer and closer to *(a,f(a))*. In other words, we take a limit! And the result will also be the slope of the tangent line at *(a,f(a))*.

$$\text{Instantaneous Rate of Change } = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

The *instantaneous rate of change* at *(a,f(a))* is also known as the *derivative of f(x) at x=a*, and we have a few different notations for this *derivative.*

$$\text{Derivative} =$$

$$\text{Instantaneous Rate of Change} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= f'(a) = \left.\frac{df}{dx}\right|_{x=a}$$

So how do we actually evaluate a *derivative?* Well, one way to do it is numerically. Here is the procedure.

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- **Enter this expression into your calculator**

- **Evaluate the limit numerically as you did before**

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$$f(x) = x^3 - x$$

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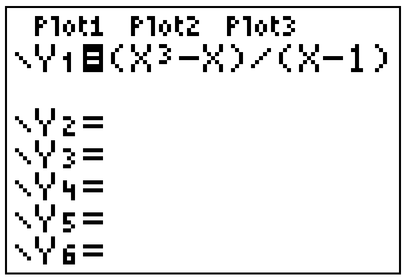

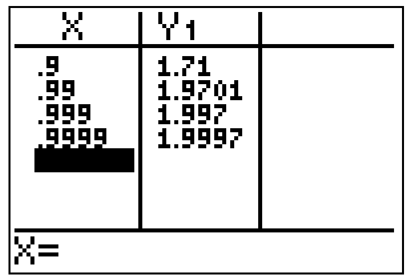

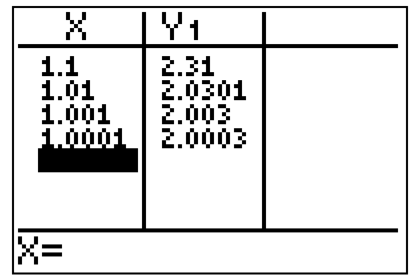

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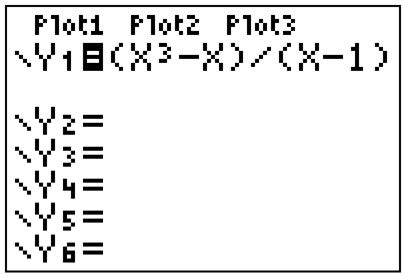
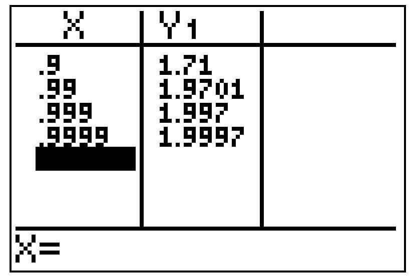
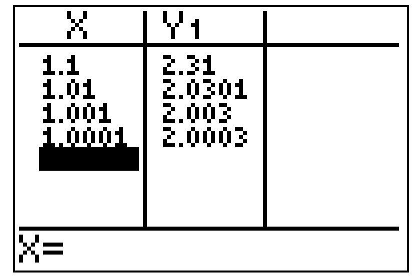






$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x-1} = \lim_{x \to 1} \frac{x^3 - x}{x-1} = 2$$

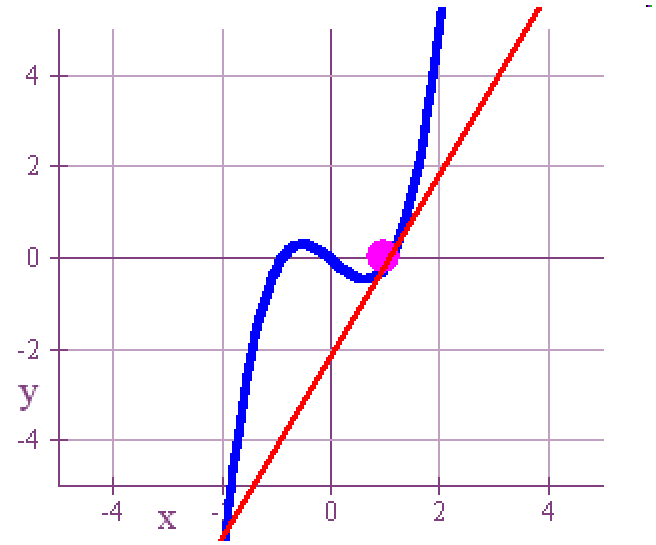
Now let's look at the graph of both the function and the tangent line.

$$f(x) = x^3 - x$$

$$f'(1) = 2$$

$$P = (1, 0)$$

$$T = 2(x-1) + 0 = 2x - 2$$



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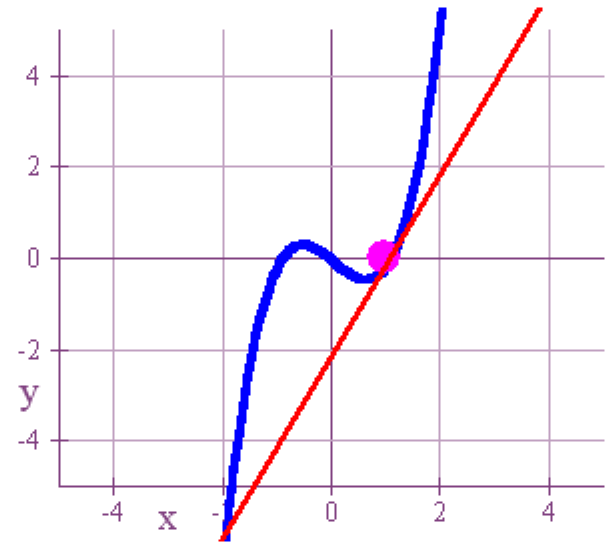

Looks good to me!

EXAMPLE:

$$f(x) = \ln x$$

$$f'(2) = ?$$

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Average Rate of Change

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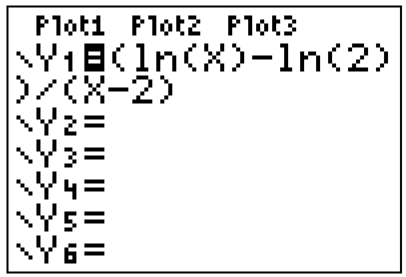

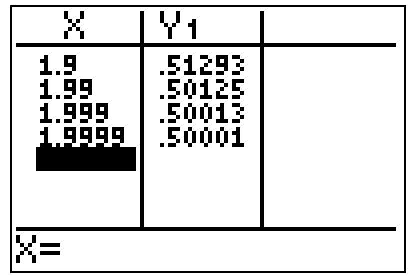

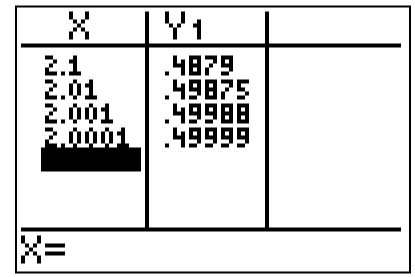

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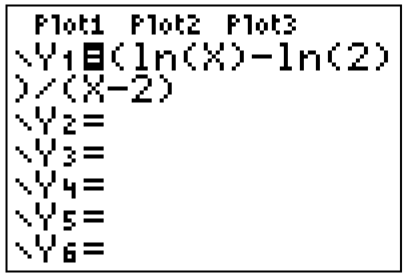
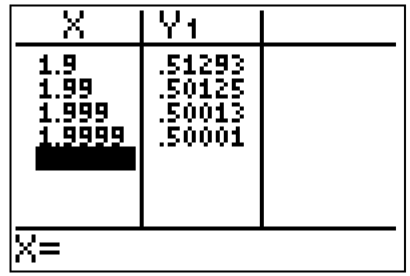
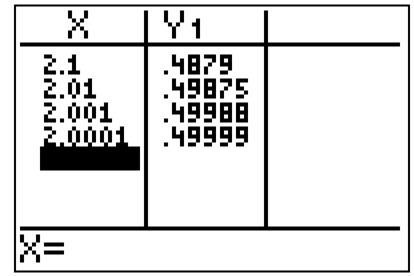






$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{\ln x - \ln 2}{x - 2} = 0.5$$

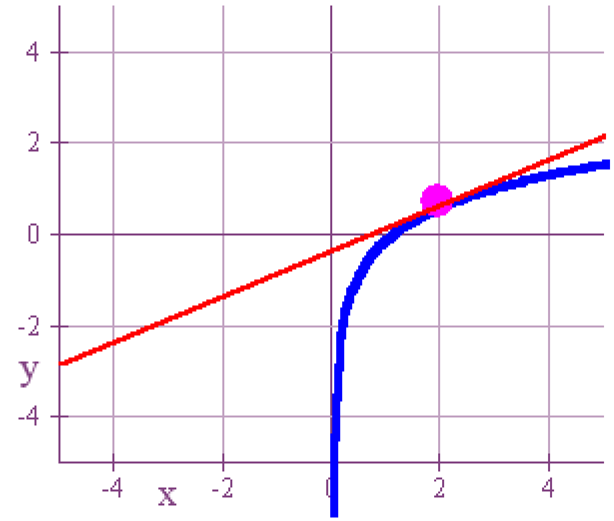
Now let's look at the graph and tangent line.

$$f(x) = \ln x$$

$$f'(2) = 0.5$$

$$P = (2, \ln 2)$$

$$T = 0.5(x - 2) + \ln 2$$



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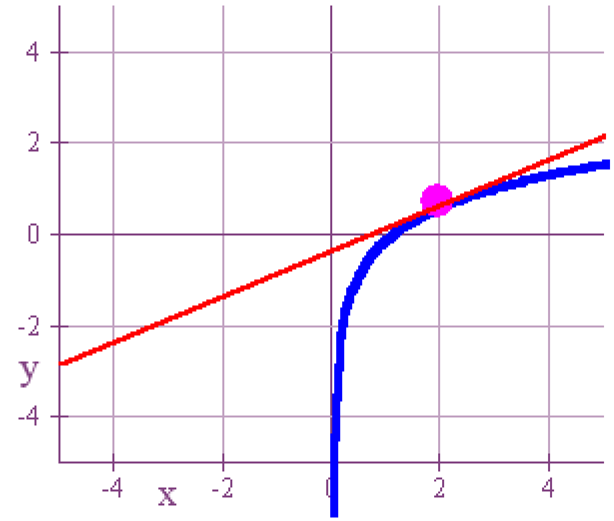

BEAUTIFUL!