HIGHER ORDER DERIVATIVES

The only thing more fun that differentiating a function once is differentiating it twice! What follow are a few examples of first and second derivatives of a function. Notice the notation we use for the second derivative!

$$y = f(x) = x^3$$

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$$\frac{dy}{dx} = f'(x) = 3x^2$$

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$$\frac{d^2y}{dx^2} = f''(x) = 6x$$

$$y = f(x) = \ln x$$

$$\frac{dy}{dx} = f'(x) = \frac{1}{x} = x^{-1}$$

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$$\frac{dy}{dx} = f'(x) = \frac{1}{x} = x^{-1}$$

$$\frac{d^2y}{dx^2} = f''(x) = -x^{-2} = -\frac{1}{x^2}$$

$$y = f(x) = 5x^2 + 3x - 2$$

$$\frac{dy}{dx} = f'(x) = 10x + 3$$

$$y = f(x) = 5x^2 + 3x - 2$$

$$\frac{dy}{dx} = f'(x) = 10x + 3$$

$$\frac{d^2y}{dx^2} = f''(x) = 10$$

$$y = f(x) = 5e^{4x}$$

$$\frac{dy}{dx} = f'(x) = 5e^{4x} \cdot 4 = 20e^{4x}$$

$$y = f(x) = 5e^{4x}$$

$$\frac{dy}{dx} = f'(x) = 5e^{4x} \cdot 4 = 20e^{4x}$$

$$\frac{d^2y}{dx^2} = f''(x) = 20e^{4x} \cdot 4 = 80e^{4x}$$

If a function represents a change in distance over time for an object, then we call the first derivative the object's velocity. The second derivative represents the object's change in velocity over time, and we call that the object's acceleration.

$$t = \text{time in seconds, } 0 \le t \le 10$$

$$y = f(t) = t^2 - 10t = \text{height above ground in feet}$$

$$v(t) = \frac{dy}{dt} = f'(t) = 2t - 10 \quad \frac{\text{feet}}{\text{second}}$$

t = time in seconds, $0 \le t \le 10$

$y = f(t) = t^2 - 10t$ = height above ground in feet

$$v(t) = \frac{dy}{dt} = f'(t) = 2t - 10 \ \frac{\text{feet}}{\text{second}}$$

$$a(t) = \frac{d^2y}{dt^2} = f''(t) = 2 \ \frac{\text{feet/second}}{\text{second}} = 2 \ \frac{\text{feet}}{\text{second}^2}$$

Suppose that the function below tells us how many video games were sold after *t* months How are sales accelerating after 10 months?

$$t = \text{time in months}, \ t \geq 0$$

$$y = f(t) = 20e^{0.4t} \ = \# \text{ of games sold}$$

$$v(t) = \frac{dy}{dt} = f'(t) = 20e^{0.4t} \cdot 0.4 = 8e^{0.4t} \ \frac{\text{games sold}}{\text{month}}$$

t = time in months, $t \geq 0$

$y = f(t) = 20e^{0.4t}$ = # of games sold

$$v(t) = \frac{dy}{dt} = f'(t) = 20e^{0.4t} \cdot 0.4 = 8e^{0.4t} \ \frac{\text{games sold}}{\text{month}}$$

$$a(t) = \frac{d^2y}{dt^2} = f''(t) = 3.2e^{0.4t} \ \frac{\text{games sold/month}}{\text{month}}$$

$$= 3.2e^{0.4t} \ \frac{\text{games sold}}{\text{month}^2}$$

t = time in months, $t \geq 0$

$y = f(t) = 20e^{0.4t}$ = # of games sold

$$v(t) = \frac{dy}{dt} = f'(t) = 20e^{0.4t} \cdot 0.4 = 8e^{0.4t} \ \frac{\text{games sold}}{\text{month}}$$

$$a(t) = \frac{d^2y}{dt^2} = f''(t) = 3.2e^{0.4t} \ \frac{\text{games sold}/\text{month}}{\text{month}}$$

$$= 3.2e^{0.4t} \ \frac{\text{games sold}}{\text{month}^2}$$

$$a(10) = \left.\frac{d^2y}{dt^2}\right|_{t=10} = f''(10) = 3.2e^4 \approx 175 \ \frac{\text{games sold}}{\text{month}^2}$$