IMPLICIT DIFFERENTIATION

$$y^2 + y = 2x$$

$$\frac{dy}{dx} = ?$$

Sometimes we have an equation that can't be explicitly solved for *y* in terms of *x*.

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In that case, how do we find the derivative of *y* with respect to *x*?

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Simple! We just go ahead and differentiate each side of the equation with respect to *x*, and then we algebraically solve for the derivative *dy/dx.*

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Since our original equation is not explicitly solved for *y* in terms of *x*, this technique is called *implicit differentiation.*

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$$\Rightarrow \frac{dy}{dx} = -\frac{1}{5y^4 + 1}$$

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$$\Rightarrow \left.\frac{dy}{dx}\right|_{\left(1/\sqrt{2},1/\sqrt{2}\right)} = -1 \Rightarrow \frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} + b \Rightarrow b = \frac{2}{\sqrt{2}}$$

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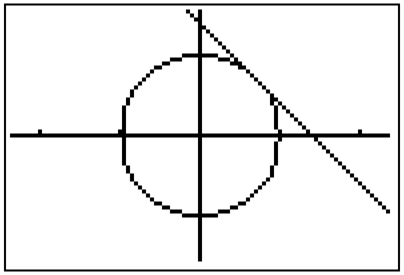

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$$\Rightarrow \frac{dy}{dx} = \frac{y}{\frac{1}{y} - x} \Rightarrow \left.\frac{dy}{dx}\right|_{(0,1)} = \frac{1}{\frac{1}{1} - 0} = 1 = slope$$

And now for a word problem from the book!