IMPROPER INTEGRALS

$$\int_{a}^{\infty} f(x)\,dx, \quad \int_{-\infty}^{b} f(x)\,dx, \quad \int_{-\infty}^{\infty} f(x)\,dx$$

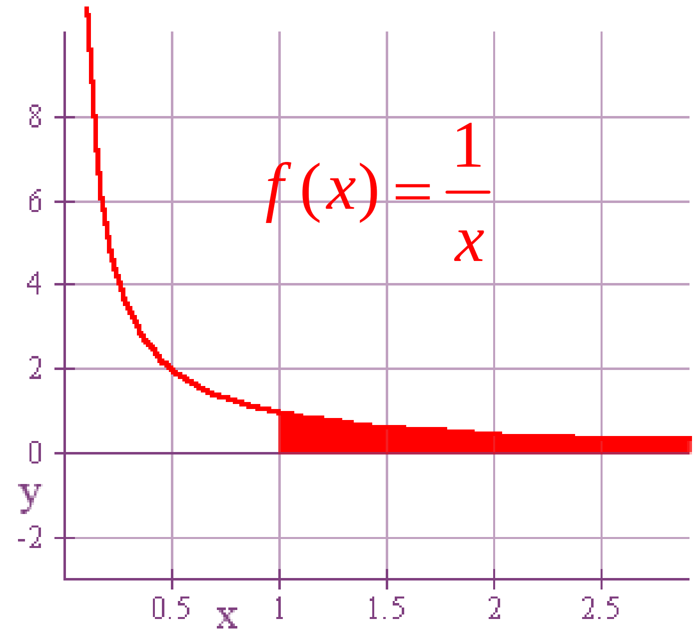
Integral that have plus or minus infinity as a limit of integration are known as *improper integrals*.

$$\int_{a}^{\infty} f(x)\,dx, \quad \int_{-\infty}^{b} f(x)\,dx, \quad \int_{-\infty}^{\infty} f(x)\,dx$$

Here are a few examples illustrating how we can evaluate improper integrals by using limits.

$$\int_{a}^{\infty} f(x)\,dx, \quad \int_{-\infty}^{b} f(x)\,dx, \quad \int_{-\infty}^{\infty} f(x)\,dx$$

What is the area under the graph of $f(x) = \dfrac{1}{x}$ and above the x-axis from $x = 1$ to $x = \infty$?



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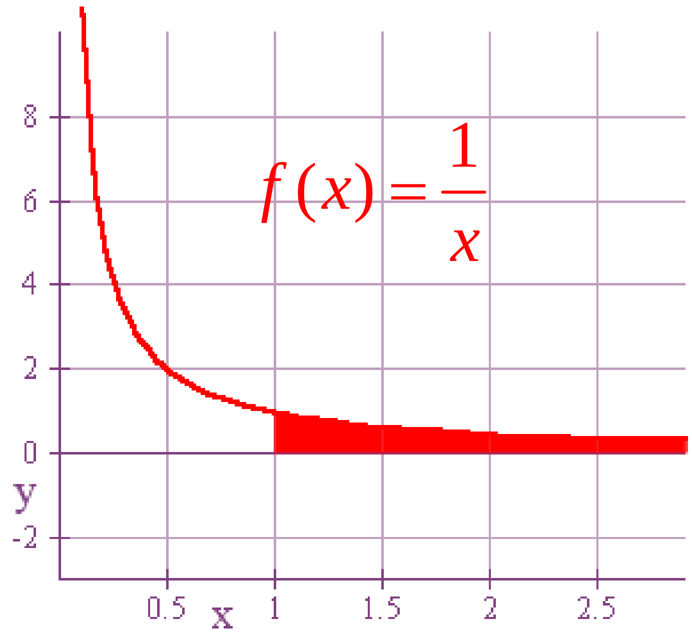
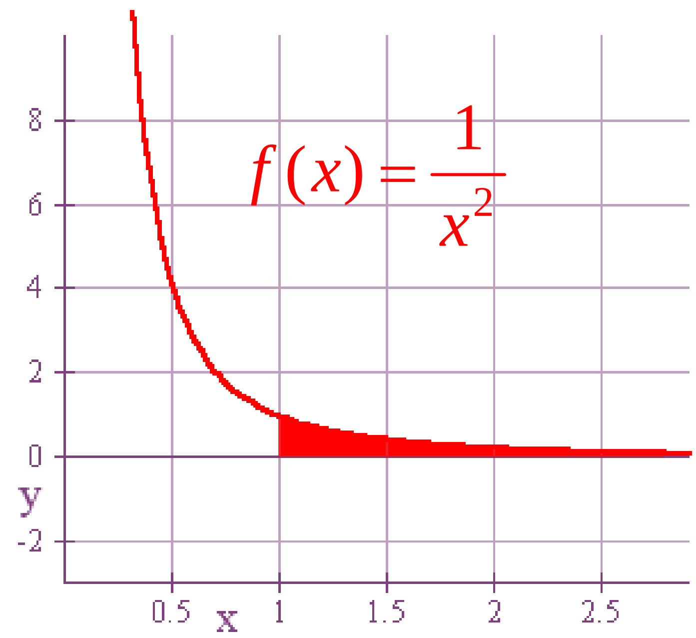
$$\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x}\,dx = \lim_{t\to\infty} \ln|x|\Big|_1^t$$

$$= \lim_{t\to\infty}\left(\ln t - \ln 1\right) = \lim_{t\to\infty} \ln t = \infty$$



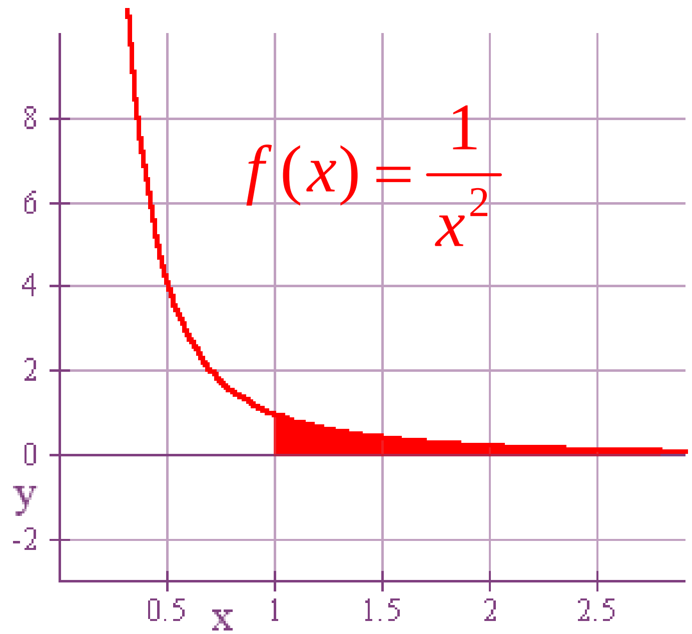
What is the area under the graph of $f(x)=\dfrac{1}{x^2}$ and above the x-axis from $x=1$ to $x=\infty$?



What is the area under the graph of $f(x) = \frac{1}{x^2}$ and above the x-axis from $x = 1$ to $x = \infty$?

$$\int_1^{\infty} \frac{1}{x^2}\,dx = \lim_{t\to\infty} \int_1^{t} \frac{1}{x^2}\,dx = \lim_{t\to\infty} -\frac{1}{x}\Bigg|_1^t$$

$$= \lim_{t\to\infty}\left(-\frac{1}{t} - \frac{-1}{1}\right) = \lim_{t\to\infty}\left(-\frac{1}{t} + 1\right) = 1$$



From 2006 to 2008 the number of new homes sold in the United States decreased dramatically. Suppose new home sales can be modeled by the function

$f(t) = 1.05e^{-0.376t}$ million homes per year where $t = 0$ corresponds to 2006. If this trend continues, then what is the total number of homes that will be sold from 2006 on?

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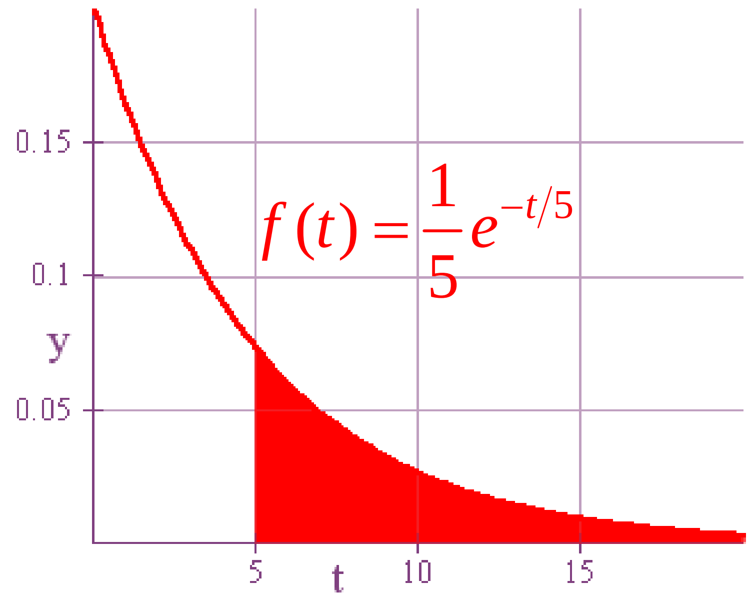
$$\int_0^{\infty} 1.05e^{-0.376t}\,dt = \lim_{n\to\infty}\int_0^{n} 1.05e^{-0.376t}\,dt = \lim_{n\to\infty}\left.\frac{1.05e^{-0.376t}}{-0.376}\right|_0^n$$

$$= \lim_{n\to\infty}\left.\frac{1.05}{-0.376e^{0.376t}}\right|_0^n = \lim_{n\to\infty}\left(\frac{1.05}{-0.376e^{0.376n}} - \frac{1.05}{-0.376}\right)$$

$$= \frac{1.05}{0.376} \approx 2.79 \text{ million homes}$$

Suppose you go to a pharmacy to pick up a prescription, and the average waiting time is 5 minutes. It can be shown that the probability that you will have to wait between a and b minutes is given by the integral

$\int_a^b \frac{1}{5} e^{-t/5}\, dt$. What is the probability that you will have to wait more than 5 minutes?



$$\int_5^\infty \frac{1}{5}e^{-t/5}\,dt = \lim_{n\to\infty}\int_5^n \frac{1}{5}e^{-t/5}\,dt = \lim_{n\to\infty} -e^{-t/5}\Big|_5^n$$

$$= \lim_{n\to\infty}\left(-e^{-n/5} - \left[-e^{-1}\right]\right) = \lim_{n\to\infty}\left(-\frac{1}{e^{n/5}} + \frac{1}{e}\right)$$

$$= \frac{1}{e} \approx 0.37$$