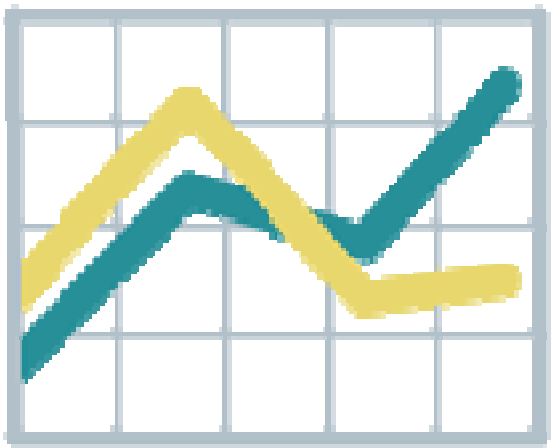
Finding Limits Graphically



Recall how we defined the limit of a function in our last presentation.

Recall how we defined the limit of a function in our last presentation.

Let $f(x)$ be a function. Then the limit of $f(x)$ as x approaches a equals L is defined as follows:

$\lim_{x \to a} f(x) = L$ means that as x gets close to, but not equal to, a, the values of $f(x)$ get closer and closer to L.

When we talk about the *limit of f(x) as x approaches a*, there are actually two main ways in which we can let *x approach a.*

Let $f(x)$ be a function. Then the limit of $f(x)$ as x approaches a equals L is defined as follows:

$\lim_{x \to a} f(x) = L$ means that as x gets close to, but not equal to, a, the values of $f(x)$ get closer and closer to L.

When we talk about the *limit of f(x) as x approaches a*, there are actually two main ways in which we can let *x approach a.*

We can approach *a* through values that are less than *a*, or we can approach *a* through values that are greater than *a.*

When we talk about the *limit of f(x) as x approaches a*, there are actually two main ways in which we can let *x approach a.*

We can approach *a* through values that are less than *a*, or we can approach *a* through values that are greater than *a.*

We call each approach a *one-sided limit*.

When we talk about the *limit of f(x) as x approaches a*, there are actually two main ways in which we can let *x approach a.*

We can approach *a* through values that are less than *a*, or we can approach *a* through values that are greater than *a.*

We call each approach a *one-sided limit*.

We can approach *a* either from below (the left or negative side), or we can approach *a* from above (the right or positive side).

For the general limit to exist, both one-sided limits must exist, and they must be equal.

For the general limit to exist, both one-sided limits must exist, and they must be equal.

For the rest of this presentation, we'll try to evaluate limits by studying the graphs of functions.

EXAMPLE:

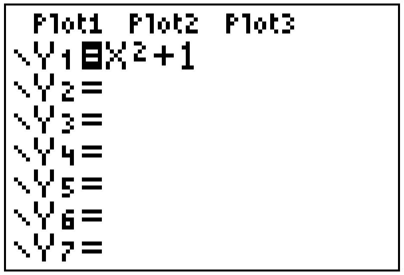
$$f(x) = x^2 + 1$$

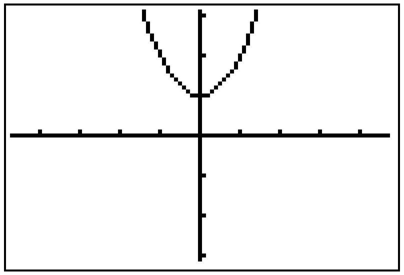
$$\lim_{x \to 0} (x^2 + 1) = ?$$

EXAMPLE:

$$f(x) = x^2 + 1$$

$$\lim_{x \to 0}(x^2 + 1) = ?$$

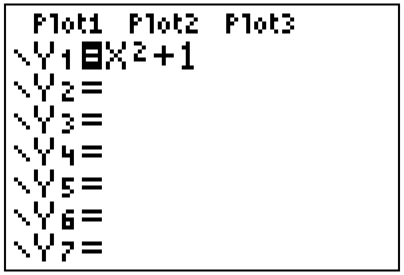


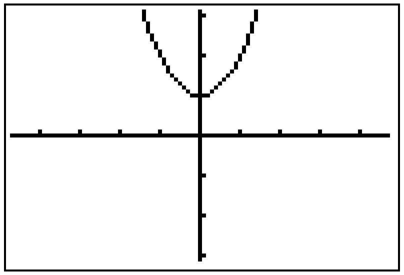


EXAMPLE:

$$f(x) = x^2 + 1$$

$$\lim_{x \to 0}(x^2 + 1) = ?$$





$$\lim_{x \to 0^-}(x^2 + 1) = 1$$

$$\lim_{x \to 0^+}(x^2 + 1) = 1$$

$$\lim_{x \to 0}(x^2 + 1) = 1$$

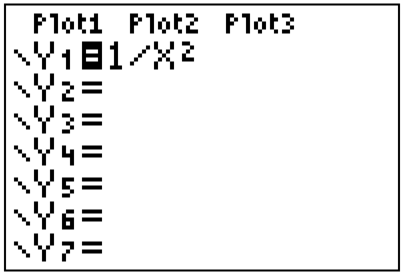
EXAMPLE:

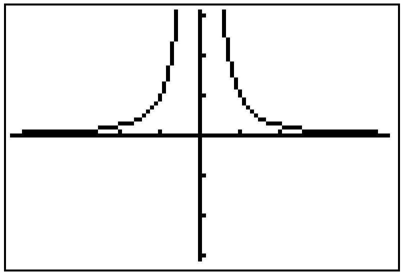
$$f(x) = \frac{1}{x^2}$$

$$\lim_{x \to 0} \left(\frac{1}{x^2} \right) = ?$$

EXAMPLE:

$$f(x)=\frac{1}{x^2}$$

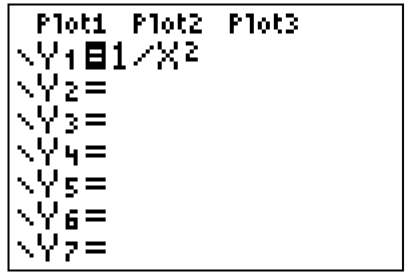


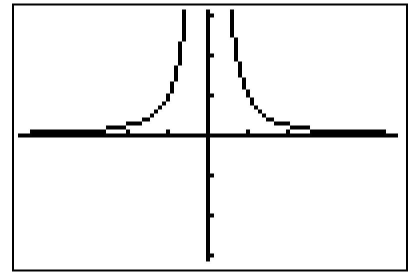


$$\lim_{x\to 0}\left(\frac{1}{x^2}\right)=?$$

EXAMPLE:

$$f(x) = \frac{1}{x^2}$$





$$\lim_{x \to 0} \left(\frac{1}{x^2} \right) = ?$$

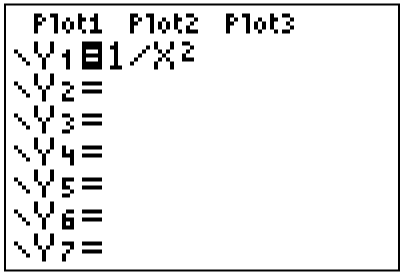
$$\lim_{x \to 0^-} \left(\frac{1}{x^2} \right) = \infty$$

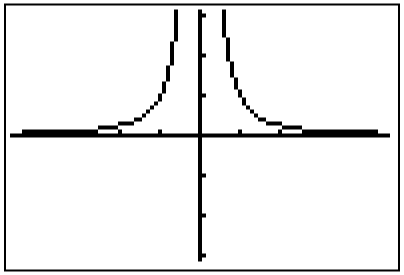
$$\lim_{x \to 0^+} \left(\frac{1}{x^2} \right) = \infty$$

$$\lim_{x \to 0} \left(\frac{1}{x^2} \right) = \infty$$

EXAMPLE:

$$f(x) = \frac{1}{x^2}$$





$$\lim_{x\to 0}\left(\frac{1}{x^2}\right) = ?$$

$$\lim_{x\to 0^-}\left(\frac{1}{x^2}\right) = \infty$$

$$\lim_{x\to 0^+}\left(\frac{1}{x^2}\right) = \infty$$

$$\lim_{x\to 0}\left(\frac{1}{x^2}\right) = \infty$$

Note that infinity is not an actual number. By writing it, we are just explaining how the limit fails to exist.

EXAMPLE:

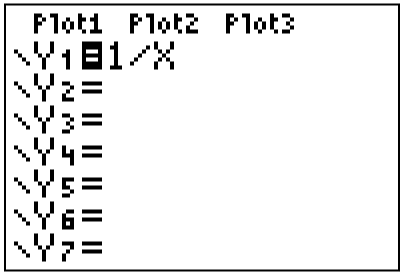
$$f(x) = \frac{1}{x}$$

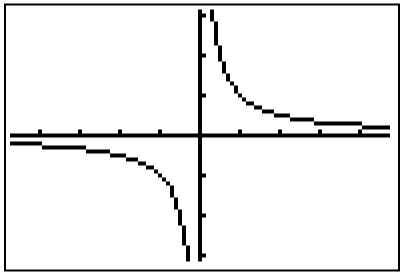
$$\lim_{x \to 0} \left(\frac{1}{x} \right) = ?$$

EXAMPLE:

$$f(x) = \frac{1}{x}$$

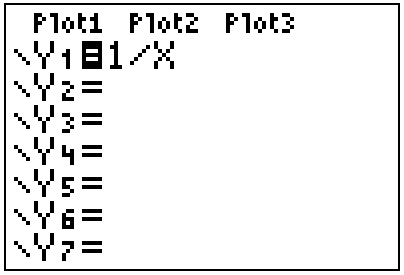
$$\lim_{x\to 0}\left(\frac{1}{x}\right) = ?$$

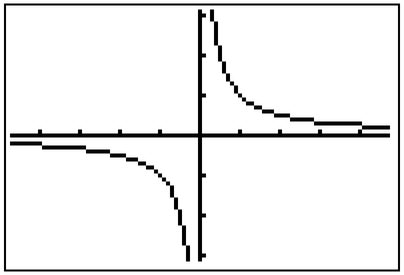




EXAMPLE:

$$f(x) = \frac{1}{x}$$





$$\lim_{x \to 0} \left(\frac{1}{x} \right) = ?$$

$$\lim_{x \to 0^-} \left(\frac{1}{x} \right) = -\infty$$

$$\lim_{x \to 0^+} \left(\frac{1}{x} \right) = \infty$$

$$\lim_{x \to 0} \left(\frac{1}{x} \right) = \text{ does not exist}$$

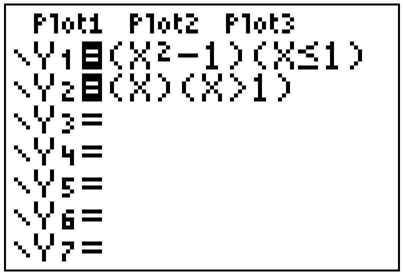
If we have a *piecewise-defined function,* there are two ways to enter it in our calculator.

$$f(x)=\begin{cases} x^2-1 & \text{if } x\le 1 \\ x & \text{if } x>1 \end{cases}$$

$$\lim_{x\to 1} f(x)=?$$

If we have a *piecewise-defined function,* there are two ways to enter it in our calculator.

$$f(x)=\begin{cases} x^2-1 & \text{if } x\le 1 \\ x & \text{if } x>1 \end{cases}$$

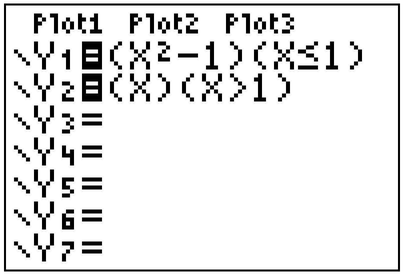


$$\lim_{x\to 1} f(x)=?$$

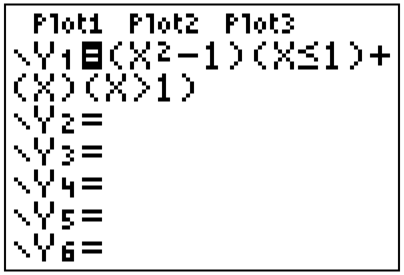
If we have a *piecewise-defined function,* there are two ways to enter it in our calculator.

$$f(x)=\begin{cases} x^2-1 & \text{if } x\le 1 \\ x & \text{if } x>1 \end{cases}$$

$$\lim_{x\to 1} f(x)=?$$

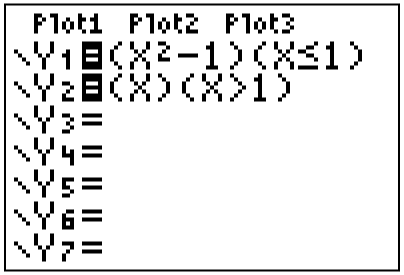


or



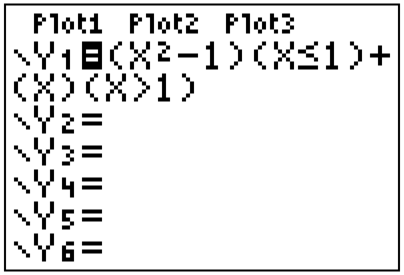
If we have a *piecewise-defined function,* there are two ways to enter it in our calculator.

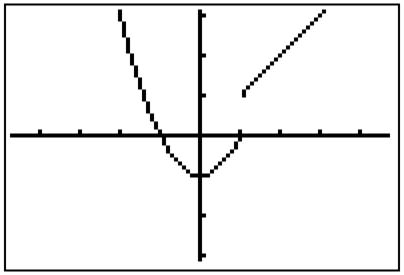
$$f(x)=\begin{cases} x^2-1 & \text{if } x\le 1 \\ x & \text{if } x>1 \end{cases}$$



$$\lim_{x\to 1} f(x)=?$$

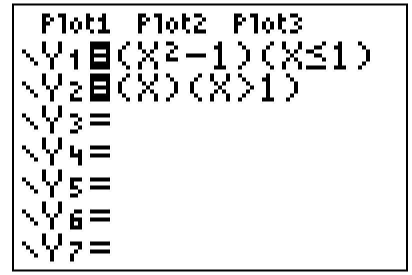
or





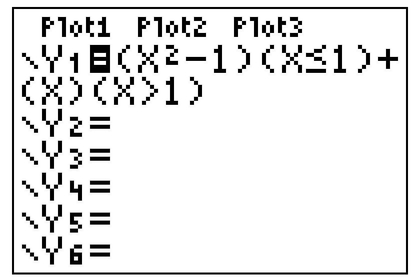
If we have a *piecewise-defined function,* there are two ways to enter it in our calculator.

$$f(x)=\begin{cases} x^2-1 & \text{if } x\le 1 \\ x & \text{if } x>1 \end{cases}$$



or

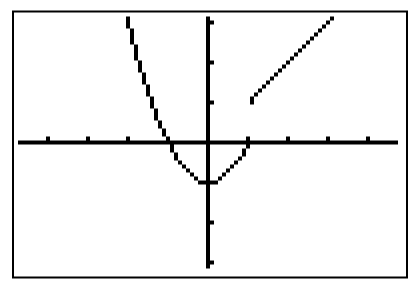
$$\lim_{x\to 1} f(x)=?$$



$$\lim_{x\to 1^-} f(x)=0$$

$$\lim_{x\to 1^+} f(x)=1$$

$$\lim_{x\to 1} f(x)= \text{ does not exist}$$



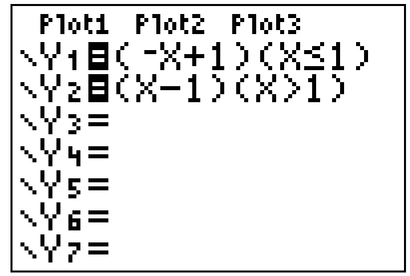
Sometimes, though, the pieces of our *piecewise -defined function* will connect, and the limit at that point will exist.

$$f(x)=\begin{cases}-x+1 & \text{if } x\le 1\\ x-1 & \text{if } x>1\end{cases}$$

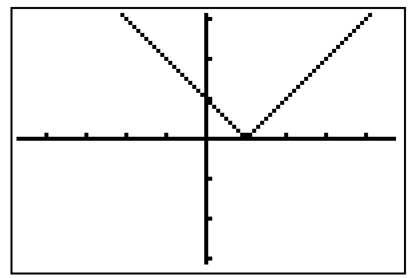
$$\lim_{x\to 1} f(x)=?$$

Sometimes, though, the pieces of our *piecewise-defined function* will connect, and the limit at that point will exist.

$$f(x)=\begin{cases}-x+1 & \text{if } x\le 1\\ x-1 & \text{if } x>1\end{cases}$$

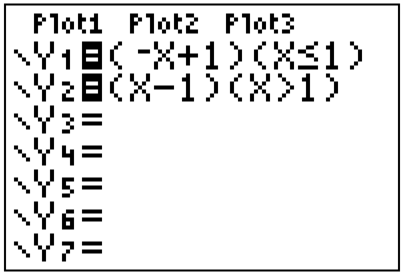


$$\lim_{x\to 1} f(x)=?$$

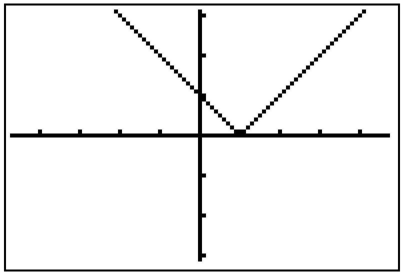


Sometimes, though, the pieces of our ***piecewise-defined function*** **will connect, and the limit at that point will exist.**

$$f(x)=\begin{cases}-x+1 & \text{if } x\le 1\\ x-1 & \text{if } x>1\end{cases}$$



$$\lim_{x\to 1} f(x)=?$$



$$\lim_{x\to 1^-} f(x)=0$$

$$\lim_{x\to 1^+} f(x)=0$$

$$\lim_{x\to 1} f(x)=0$$

Math is Cool!