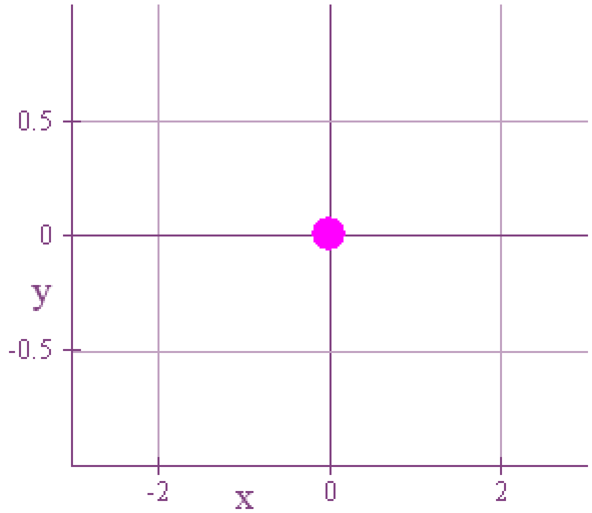
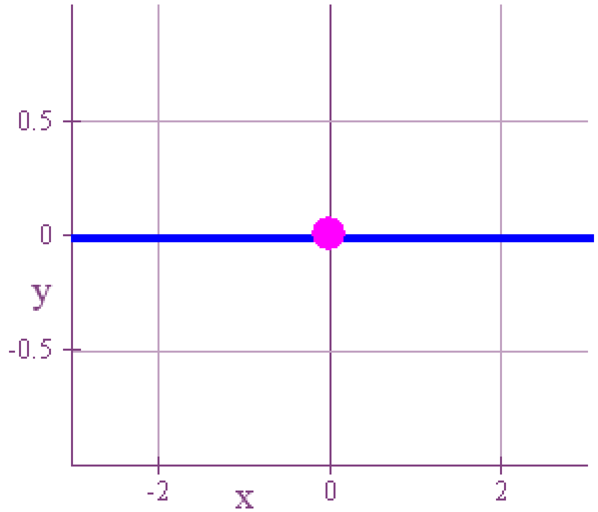
THE SECOND DERIVATIVE TEST

Suppose you have a point on the graph of some function, and suppose that the first derivative is zero at this point.

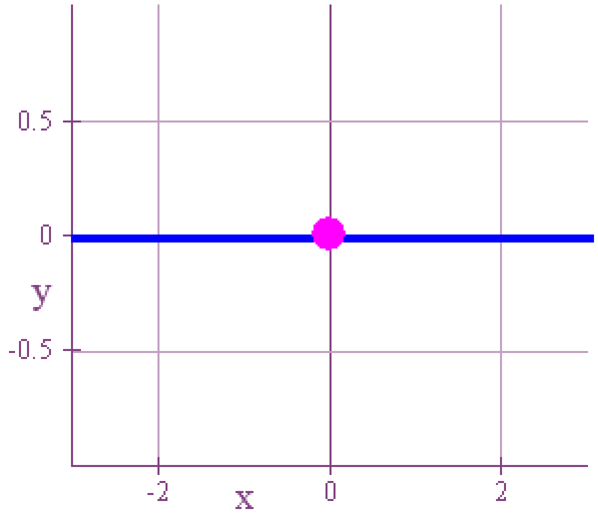


In that case, there will be a horizontal tangent line at this point.



$$f'(x) = 0$$

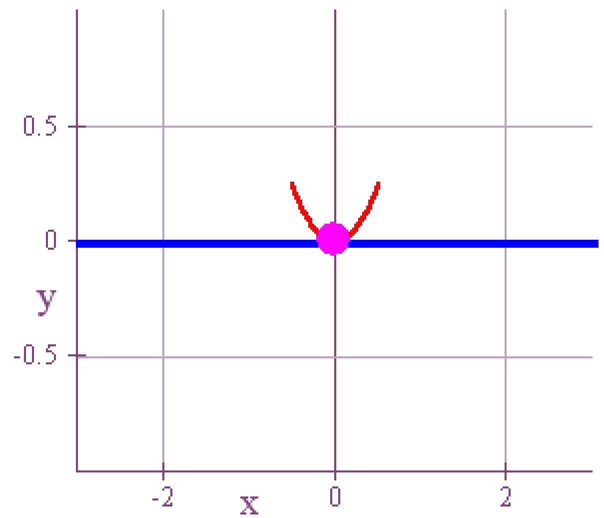
Now suppose that we also know that the second derivative is positive at this point.



$$f'(x) = 0$$

$$f''(x) > 0$$

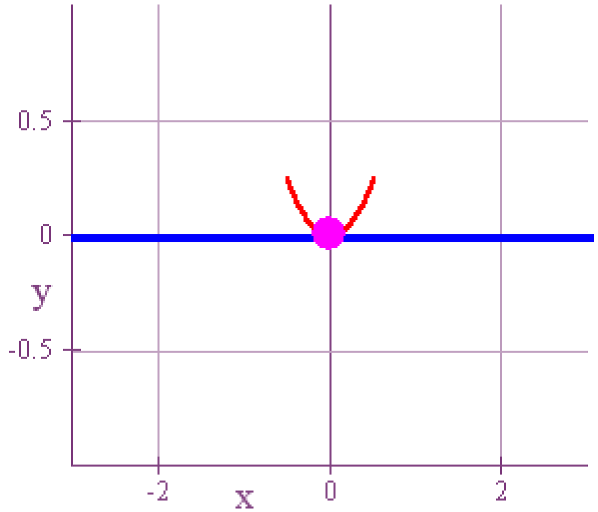
Then that means that the graph will also be concave up at this point.



$$f'(x) = 0$$

$$f''(x) > 0$$

Taken together, these two facts mean that we will have a relative minimum value at our point *x*.

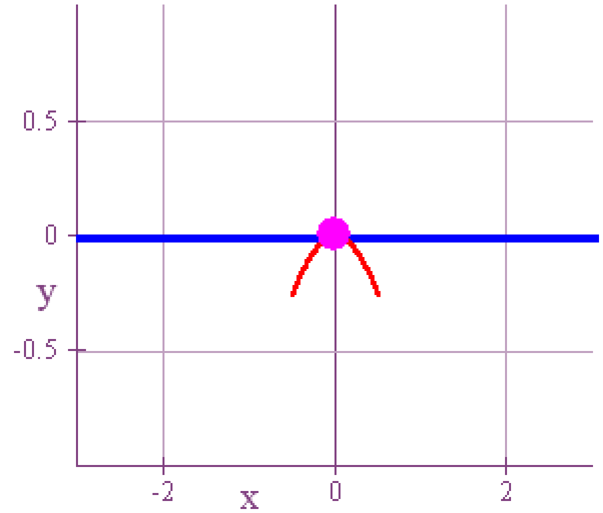


$$f'(x) = 0$$

$$f''(x) > 0$$

$f(x)$ is a relative minimum

Similarly, if the first derivative was zero and the second derivative was negative, then we would have a relative maximum value at our point *x*.



$$f'(x) = 0$$

$$f''(x) < 0$$

$f(x)$ is a relative maximum

Example:

$$f(x) = x^3 - 3x$$

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$$f'(x) = 0 \Rightarrow x = -1 \text{ or } x = 1$$

$$f''(x) = 6x$$

$$f''(-1) = -6 \Rightarrow f(-1) = 2 \text{ is a relative maximum}$$

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$$f(x) = x^3 - 3x$$

$$f'(x) = 3x^2 - 3 = 3(x+1)(x-1)$$

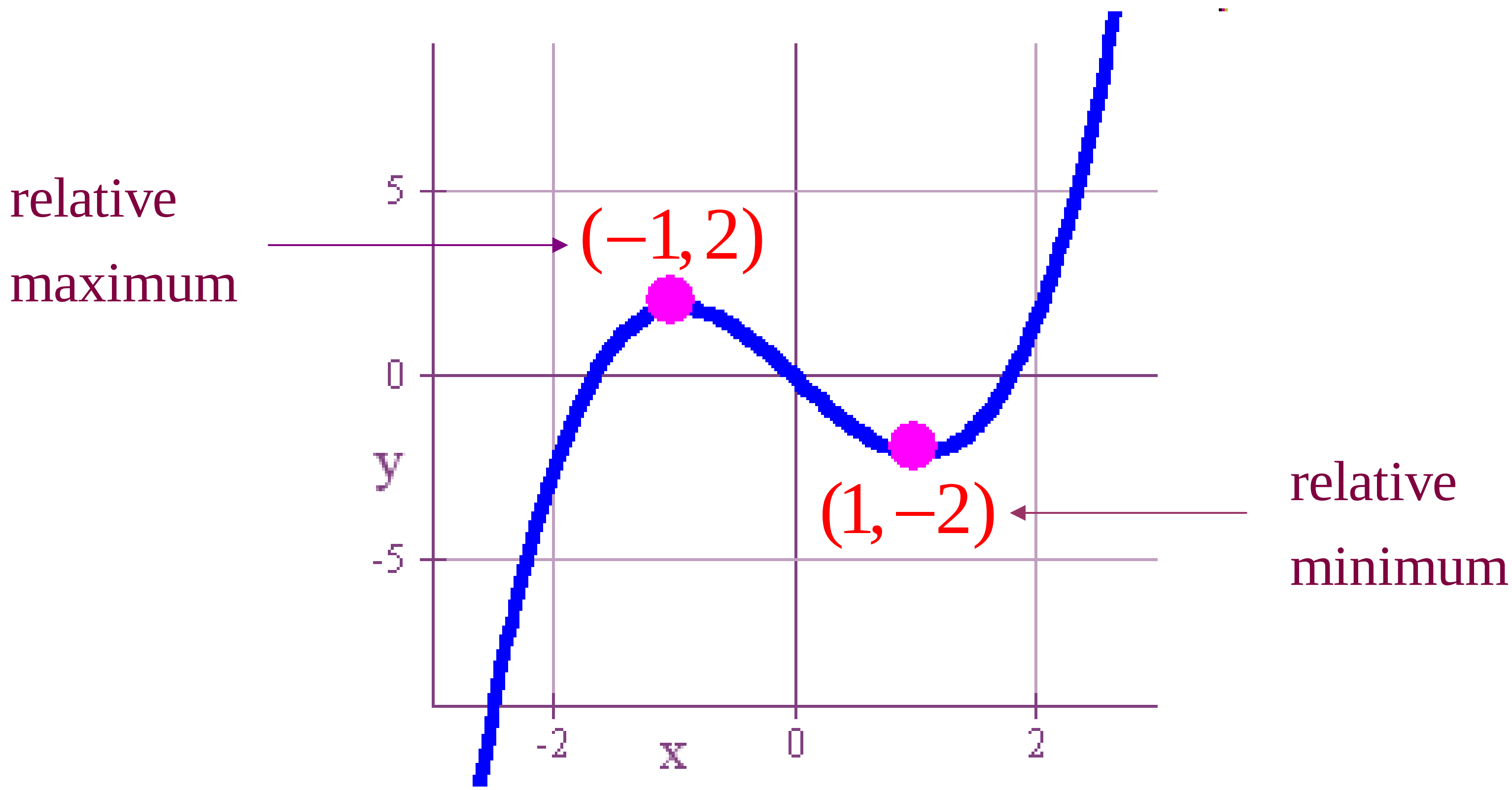
$$f'(x) = 0 \Rightarrow x = -1 \text{ or } x = 1$$

$$f''(x) = 6x$$

$$f''(-1) = -6 \Rightarrow f(-1) = 2 \text{ is a relative maximum}$$

$$f''(1) = 6 \Rightarrow f(1) = -2 \text{ is a relative minimum}$$

$$f(x) = x^3 - 3x$$



The Second Derivative Test:

1. If $y = f(x)$ is a function and if at some point a, $f'(a) = 0$ and $f'' > 0$, then $f(a)$ is a relative minimum.

2. If $y = f(x)$ is a function and if at some point a, $f'(a) = 0$ and $f'' < 0$, then $f(a)$ is a relative maximum.