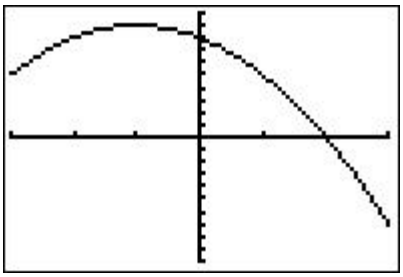
ABSOLUTE EXTREMA EXERCISES – ANSWERS

For each problem below, identify the coordinates of all absolute extrema on the given domain, and label each as either an absolute maximum or an absolute minimum. Give exact answers in fraction form

1. $f(x) = -x^2 - 2x + 8, \ [-3,3]$



$f'(x) = -2x - 2 = -2(x+1)$

$f'(x) = 0 \Rightarrow -2(x+1) = 0 \Rightarrow x = -1$ critical point

$f(-3) = 5$

$f(-1) = 9$ is an absolute maximum

$(-1,9)$ is an absolute maximum point

$f(3) = -7$ is an absolute minimum

$(3,-7)$ is an absolute minimum point

2. $f(x) = x^2 - 6x + 1, \ [-4,0]$

$f'(x) = 2x - 6 = 2(x-3)$

$f'(x) = 0 \Rightarrow 2(x-3) = 0 \Rightarrow x = 3$ critical point

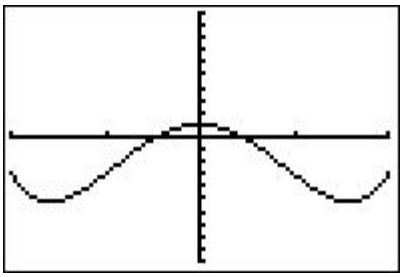
$f(-4) = 41$ is an absolute maximum

$(-4,41)$ is an absolute maximum point

$f(0) = 1$ is an absolute minimum

$(0,1)$ is an absolute minimum point

3. $f(x) = x^4 - 5x^2 + 1, \ [-2,2]$



$f'(x) = 4x^3 - 10x = 2x(2x^2 - 5)$

$f'(x) = 0 \Rightarrow x = 0$ or $2x^2 - 5 = 0$

$\Rightarrow x = 0$ or $x = \pm\sqrt{\frac{5}{2}} = \frac{\pm\sqrt{10}}{2}$ critical points

$f(-2) = -3$

$f(-\sqrt{5/2}) = -21/4$ is an absolute minimum

$(-\sqrt{5/2}, -21/4)$ is an absolute minimum point

$f(0) = 1$ is an absolute maximum

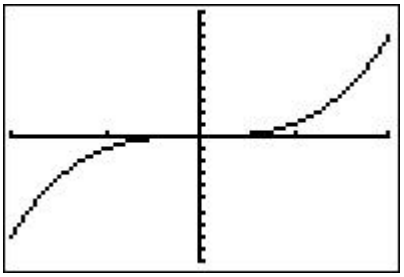
$(0,1)$ is an absolute maximum point

$f(\sqrt{5/2}) = -21/4$ is an absolute minimum

$(\sqrt{5/2}, -21/4)$ is an absllute minimum point

$f(2) = -3$

4. $f(x) = x^3, \ [-2,2]$



$f'(x) = 3x^2$

$f'(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0$ critical point

$f(-2) = -8$ is an absolute minimum

$(-2,-8)$ is an absolute minimum point

$f(0) = 0$

$f(2) = 8$ is an absolute maximum

$(2,8)$ is an absolute maximum point

5. $f(x) = x^3 - x^2 - x - 1, \ [-1, 2]$

$f'(x) = 3x^2 - 2x - 1 = (3x + 1)(x - 1)$

$f'(x) = 0 \Rightarrow 3x + 1 = 0$ or $x - 1 = 0$

$\Rightarrow x = -1/3$ or $x = 1$ critical points

$f(-1) = -2$ is an absolute minimum

$(-1, -2)$ is an absolute minimum point

$f(-1/3) = -22/27$

$f(1) = -2$ is an absolute minimum

$(1, -2)$ is an absolute minimum point

$f(2) = 1$ is an absolute maximum

$(2, 1)$ is an absolute maximum point