THE BINOMIAL DISTRIBUTION

The *binomial probability distribution* generally occurs when a procedure has two outcomes that may be categorized as either *success* or *failure*.

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- There procedure has a fixed number of trials *n.*
- The outcome of each trial is independent of the others.
- The outcome of each trial can be classified as either *success (S)* or *failure (F).*
- The probability of *success* is the same for each trial, $P(S)=p$, and the probability of *failure* is $P(F)=1-p=q$.

Suppose we take a 5-question multiple choice test and that there a 4 choices for each problem.

Even though each problem has 4 choices, we can still think of each answer as resulting in one of two outcomes*.* Our answer is either a *success* or a *failure*.

Now suppose that we guess on each question. Then the probability of *success* is $p = 1/4 = 0.25$, and the probability of *failure* is $q = 3/4 = 0.75$.

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Furthermore, our random variable is
X = *the number of successes.*

We now just need to figure out how to determine the probability of getting x successes.

Suppose that we guess the first three questions correctly and then miss the next two. The probability of this outcome is shown below.

$$P(\text{correct} \ \& \ \text{correct} \ \& \ \text{correct} \ \& \ \text{wrong} \ \& \ \text{wrong})$$

$$= (.25)(.25)(.25)(.75)(.75) = .25^3 .75^2 \approx 0.0088$$

Notice, though, that any result that involves three correct and two wrong will have exactly the same probability.

$$P(\text{correct \& wrong \& correct \& wrong \& correct})$$

$$= (.25)(.75)(.25)(.75)(.25) = .25^3 .75^2 \approx 0.0088$$

Now the question is just how many ways can we complete the test so that three of the answers are correct and two are wrong?

$$P(\text{correct \& wrong \& correct \& wrong \& correct})$$

$$= (.25)(.75)(.25)(.75)(.25) = .25^3 .75^2 \approx 0.0088$$

The answer is given by one of our counting formulas. Think of it in terms of having five answers in front of us and we want to pick exactly three of them to be *correct*. The number of ways we can do this is equal to the number of combinations of five objects choose three.

$$ {}_5C_3 = \frac{5!}{(5-3)!3!} = 10 $$

So we now know that there are 10 outcomes that contain three successes and two failures, and each individual outcome has a probability of approximately 0.0088.

$${}_5C_3 = \frac{5!}{(5-3)!3!} = 10$$

$$.25^3 .75^2 \approx 0.0088$$

Furthermore, since these outcomes are *mutually exclusive* from one another, we can find the overall probability of getting one or the other by just adding the individual probabilities together.

$$ {}_5C_3 = \frac{5!}{(5-3)!3!} = 10 $$

$$ .25^3 .75^2 \approx 0.0088 $$

In other words, the probability of getting exactly three successes is given by the formula below.

$$P(3) = {}_5C_3(.25^3)(.75^2) \approx 0.088$$

More generally, in a binomial experiment with *n* trials, the probability of getting *x* successes is:

$$P(x) = {}_nC_x (p^x)(q^{n-x})$$

We can now complete the probability distribution for this experiment.

$$P(x) = {}_nC_x(p^x)(q^{n-x})$$

$$P(0) = {}_5C_0(.25^0)(.75^5) \approx .23730$$

$$P(1) = {}_5C_1(.25^1)(.75^4) \approx .39551$$

$$P(2) = {}_5C_2(.25^2)(.75^3) \approx .26367$$

$$P(3) = {}_5C_3(.25^3)(.75^2) \approx .08789$$

$$P(4) = {}_5C_4(.25^4)(.75^1) \approx .01465$$

$$P(5) = {}_5C_5(.25^5)(.75^0) \approx .00098$$

x	*P(x)*
0	0.23730
1	0.39551
2	0.26367
3	0.08789
4	0.01465
5	0.00098

As usual, however, there is also a way to do this on our calculator.