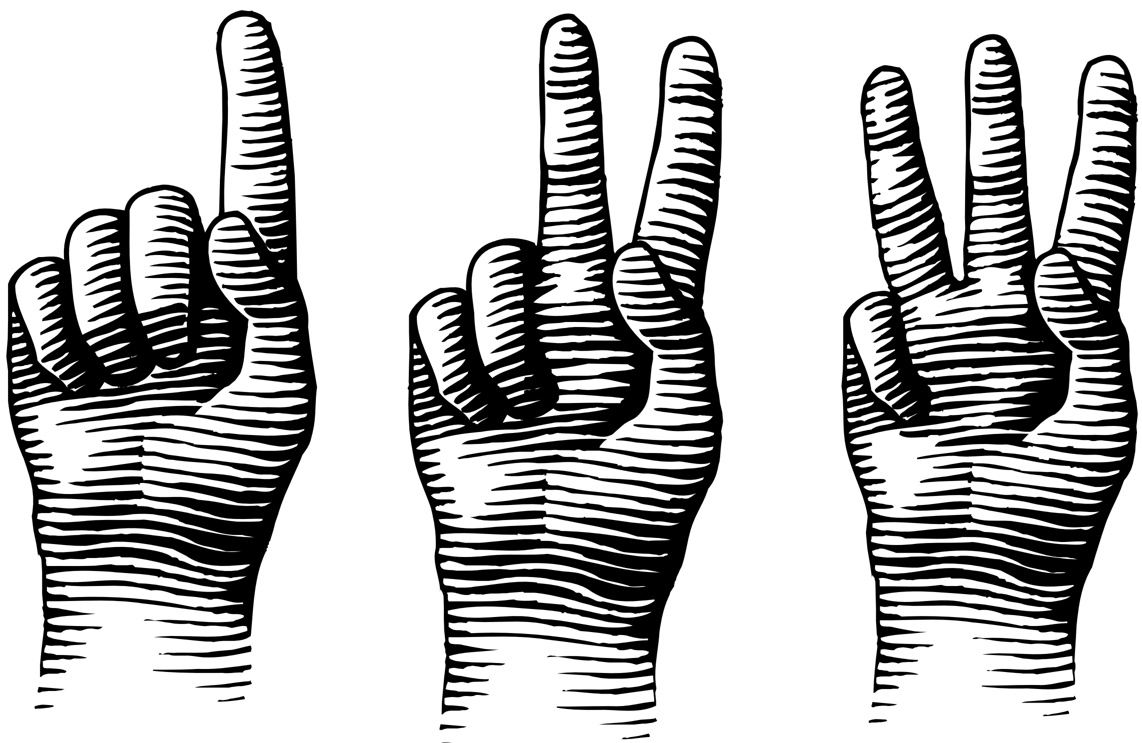
COUNTING



FUNDAMENTAL COUNTING RULE:

For a sequence of two events, if the first event can happen in *m* ways and the second event can happen in *n* ways, then together the events can happen in *m* x n *ways.*

Example:

You select one meat topping and one veggie topping for a pizza. For the meats you can get hamburger, sausage, or pepperoni, and for the veggies you can get onions or mushrooms. How many possibilities are there?

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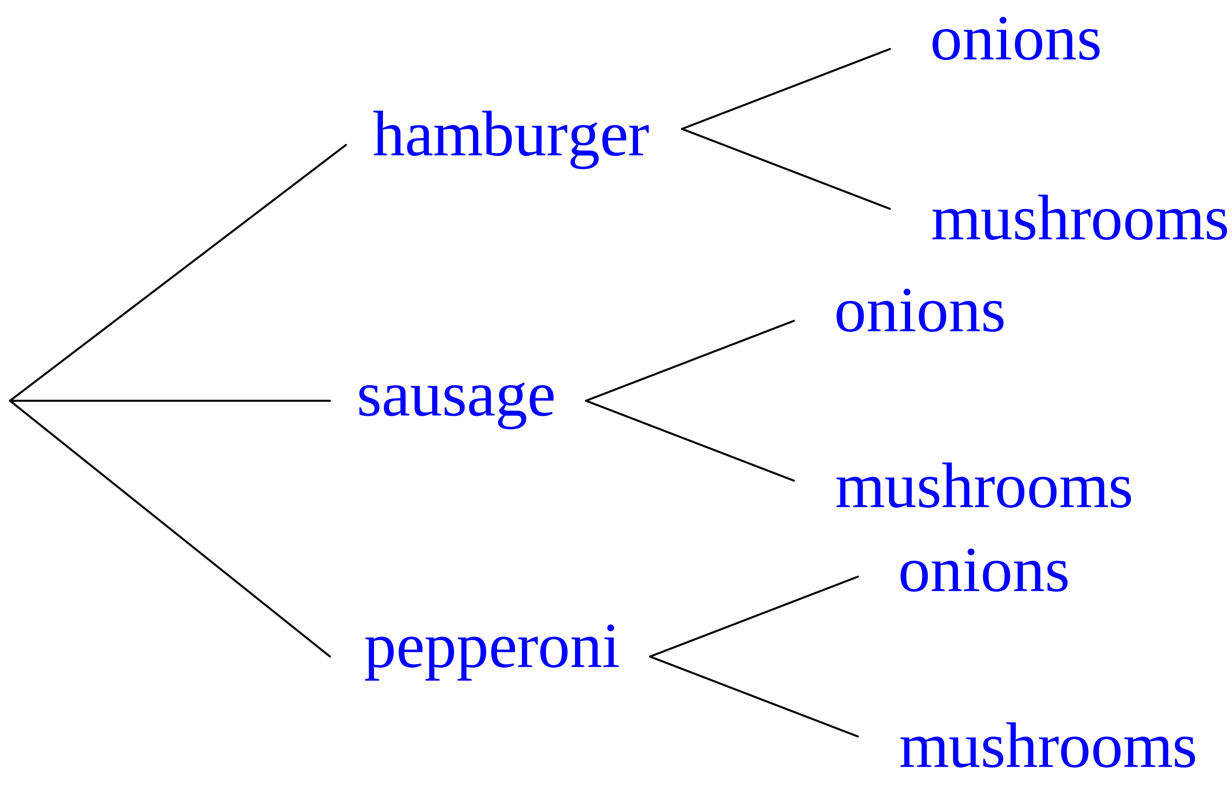

$$3 \cdot 2 = 6$$

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Should a *combination lock* be called a *permutation lock?*

How many permutations can we make of the letters in the set *A*? (draw without replacement)

$$A = \{a, b, c\}$$

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$$A = \{a, b, c\}$$

$$3 \cdot 2 \cdot 1 = 6$$

abc	bac	cab
acb	bca	cba

How many combinations are represented below?

$$A = \{a, b, c\}$$

$$\begin{array}{ccc} abc & bac & cab \\ acb & bca & cba \end{array}$$

How many combinations are representd below?

$$A = \{a, b, c\}$$

Only one!

abc	bac	cab
acb	bca	cba

Definition: The product (n)(n-1)(n-2) … (1) is called *n factorial* and denoted by *n!*.

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$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

$$\frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 5 \cdot 4 = 20$$

Below is a formula for counting the number of permutations of *n* objects if we choose only *r*.

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$${}_{n}P_{r} = \frac{n!}{(n-r)!}$$

$${}_{5}P_{2} = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 5 \cdot 4 = 20$$

Now we look at the formula for counting the number of combinations of *n* objects if we choose only *r*.

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$$ {}_{n}C_{r} = \frac{n!}{(n-r)!r!} $$

$$ {}_{5}C_{2} = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} = \frac{120}{6 \cdot 2} = 10 $$

How many different committees of 5 can we form from 20 people?

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$$_{20}C_5 = \frac{20!}{15!5!} = 15,504$$

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$$_{15}P_5 = \frac{15!}{10!} = 360,360$$

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$$5 \cdot 10 = 50$$

If a pizza can be made with 5 different meat toppings and 10 different vegetable toppings and you pick <u>two</u> of each, how many pizzas are possible?

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$$\left({}_{5}C_{2}\right)\left({}_{10}C_{2}\right)=10\cdot 45=450$$

How many different five-card poker hands are possible?

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$$_{52}C_5 = 2,598,960$$

How many different five-card hands are possible if we draw *with replacement?*
(and count permutations instead of combinations)

$$52 \cdot 52 \cdot 52 \cdot 52 \cdot 52 = 52^5 = 380,204,032$$

How many different permutations are possible of the letters in the word *meat*?

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$$4! = 24$$

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$$\frac{4!}{2!} = \frac{24}{2} = 12$$