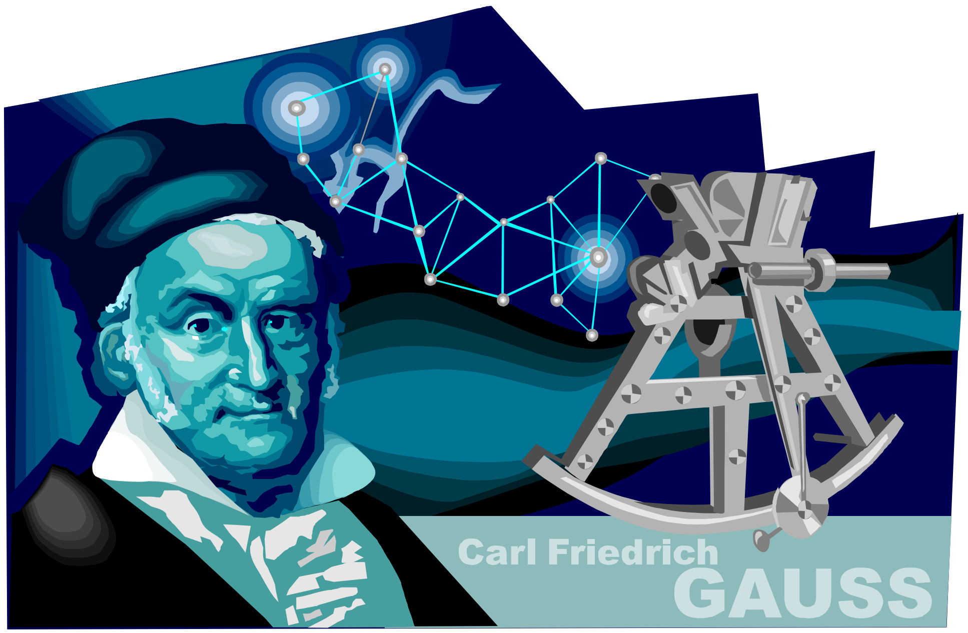
FUNCTIONS OF SEVERAL VARIABLES



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As usual, a set of specific values for the inputs always determines a specific value for the output.

Examples:

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2. $$Perimeter = 2L + 2W$$

3. $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

4. $$z = f(x, y) = x^2 + y^2$$

A function of several variables may be expressed in several different ways.

Verbally:

"The output is the sum of the squares of the two inputs."

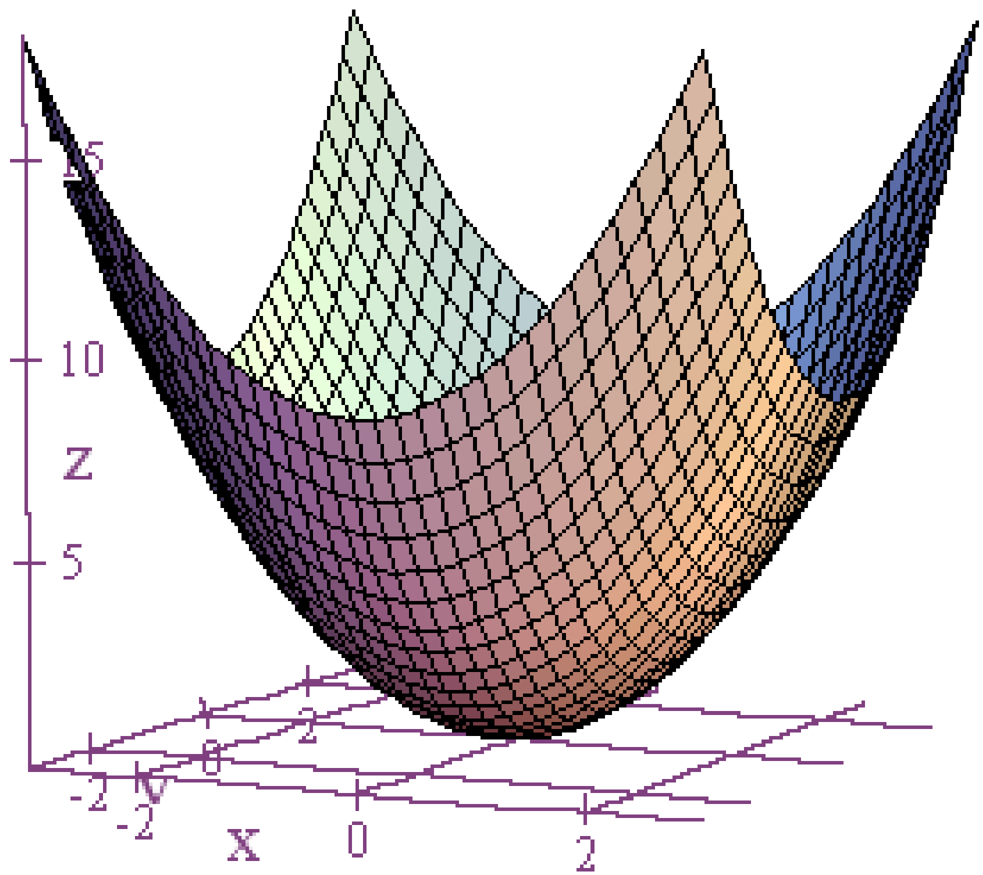
Algebraically:

$$z = f(x, y) = x^2 + y^2$$

Numerically:

y\x	-2	-1	0	1	2
-2	8	5	4	5	8
-1	5	2	1	2	5
0	4	1	0	1	4
1	5	2	1	2	5
2	8	5	4	5	8

Or Graphically:



We can evaluate a function of several variables by simply plugging in values for each of the variables present.

$$z = f(x, y) = xy^2$$

$$f(1,1) = 1 \cdot 1^2 = 1$$

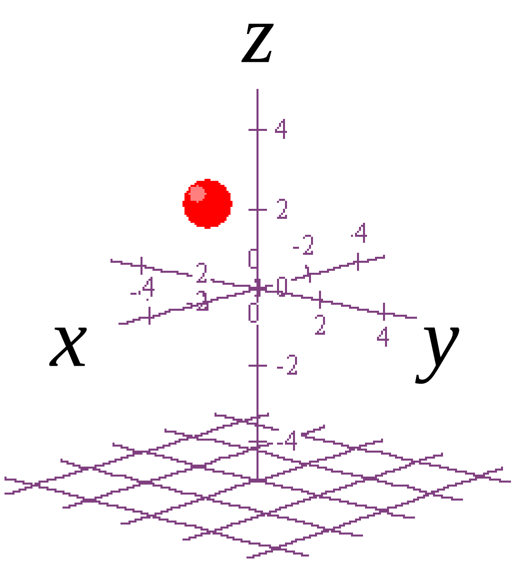
$$f(2,3) = 2 \cdot 3^2 = 18$$

$$f(4,2) = 4 \cdot 2^2 = 16$$

$$f(-3,-2) = -3 \cdot (-2)^2 = -12$$

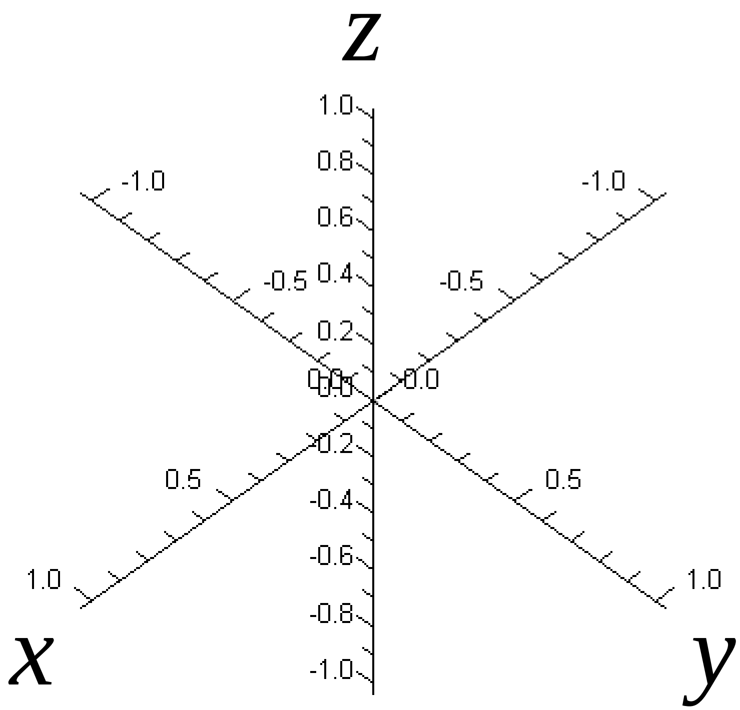
PLOTTING POINTS

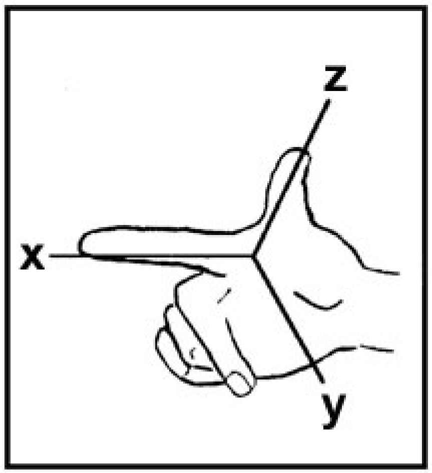
We can locate positions in 3-dimensional space by establishing an x-axis, y-axis, and z-axis, and then specifying an x-coordinate, y-coordinate, and z-coordinate for particular points.



$$(x, y, z) = (4, 2, 3)$$

This orientation called a right-hand coordinate system.



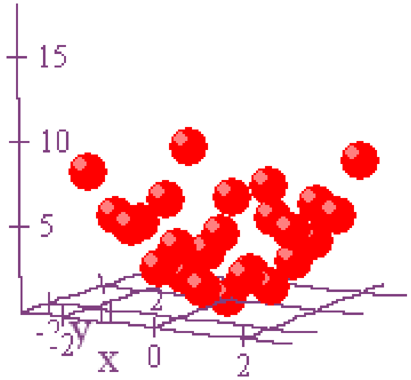


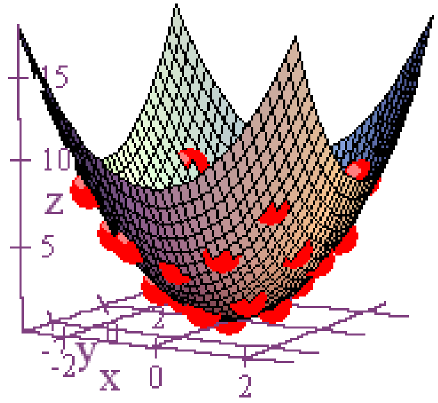
We can use the function below to generate the coordinates of points to plot.

$$z = f(x, y) = x^2 + y^2$$

y\x	-2	-1	0	1	2
-2	8	5	4	5	8
-1	5	2	1	2	5
0	4	1	0	1	4
1	5	2	1	2	5
2	8	5	4	5	8

And from there it's just a matter of plotting points until the plot thickens!

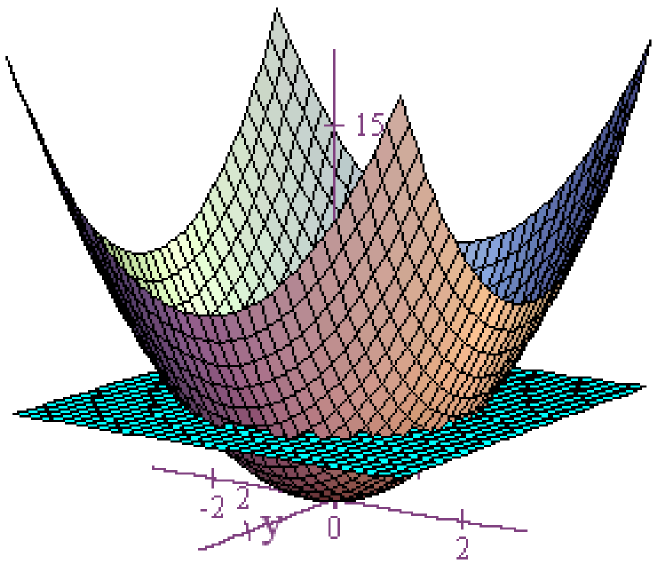


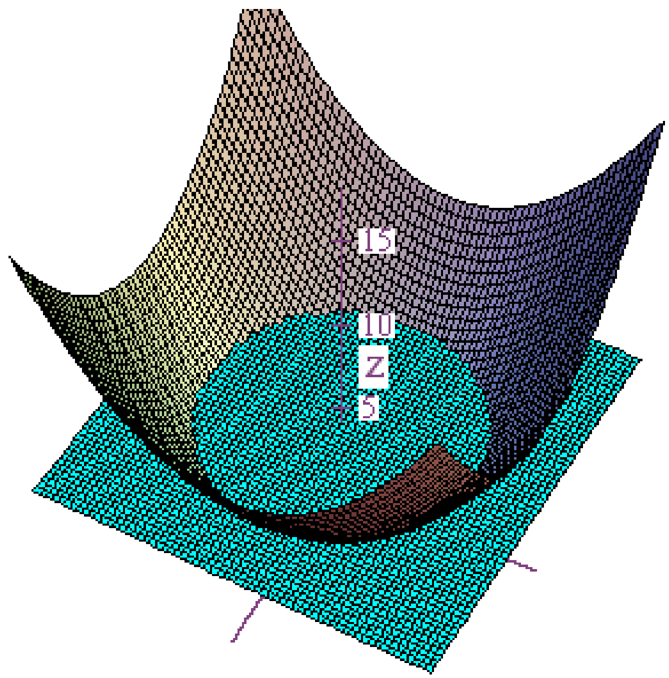


One way to analyze a function of several variables is to set the output to a fixed value and see what kind of cross-section this results in.

$$z = x^2 + y^2$$

$$z = 4$$



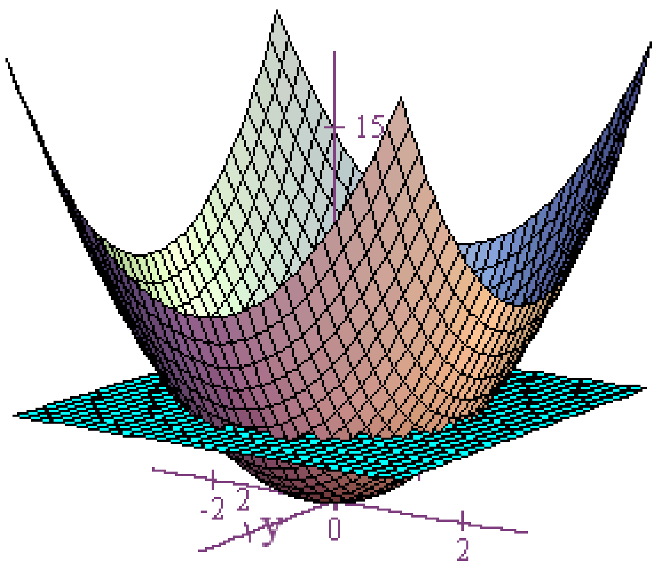


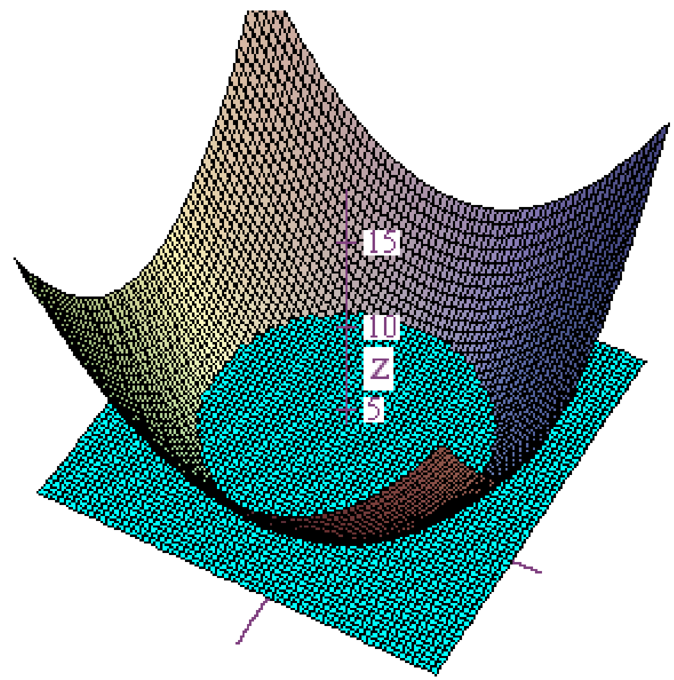
The cross-section is the circle $x^2+y^2=4$.

$$z = x^2 + y^2$$

$$z = 4$$

$$x^2 + y^2 = 4$$



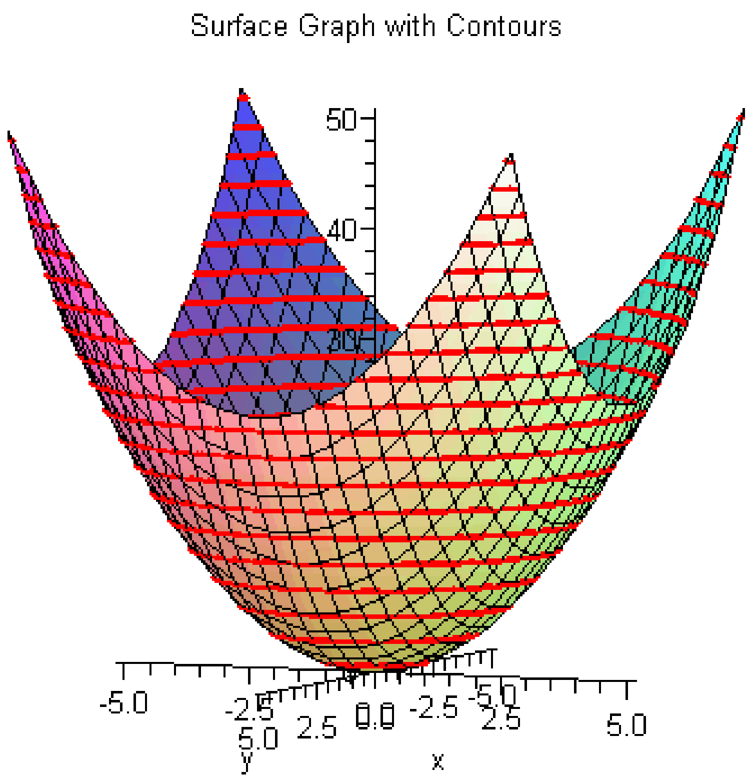


If we plot these cross-sections on the graph itself, we call them contour lines.

$$z = x^2 + y^2$$

$$z = 4$$

$$x^2 + y^2 = 4$$

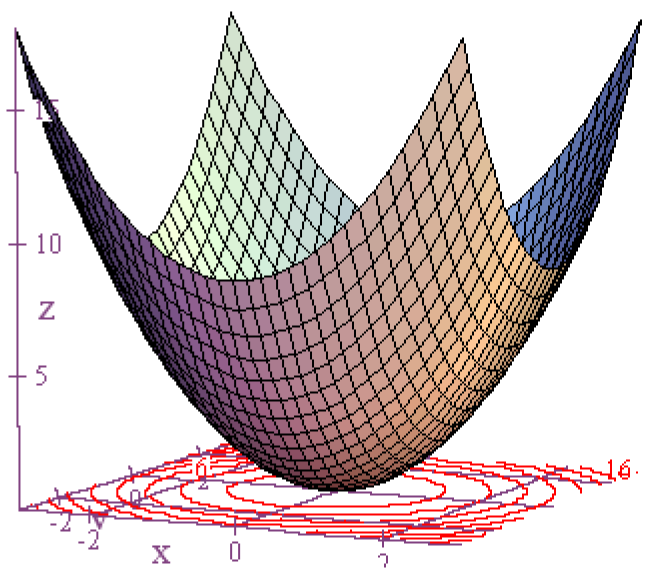


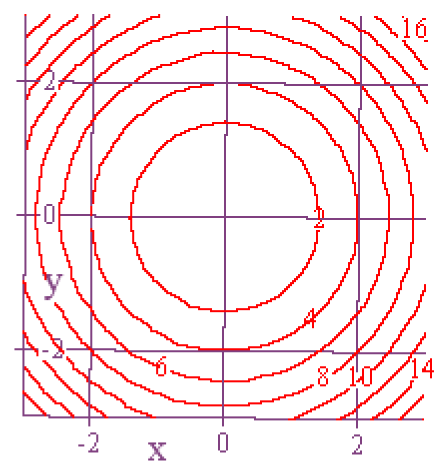
On the other hand, if we plot these cross-sections in the xy-plane, then we call them level curves.

$$z = x^2 + y^2$$

$$x^2 + y^2 = 4$$

$$z = 4$$



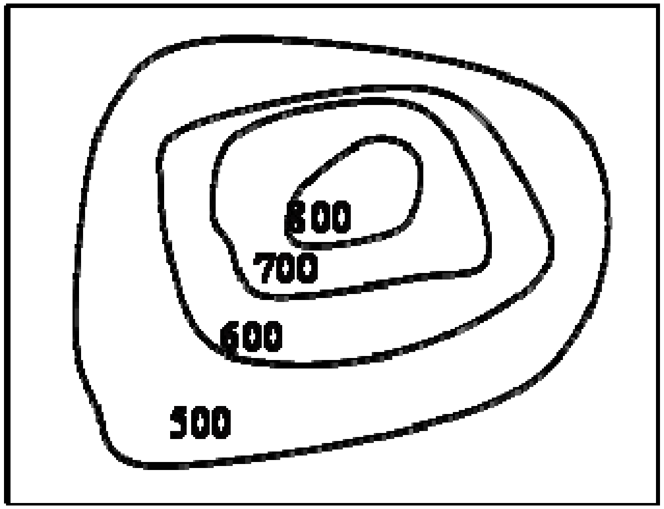


The result is essentially a topographic map.

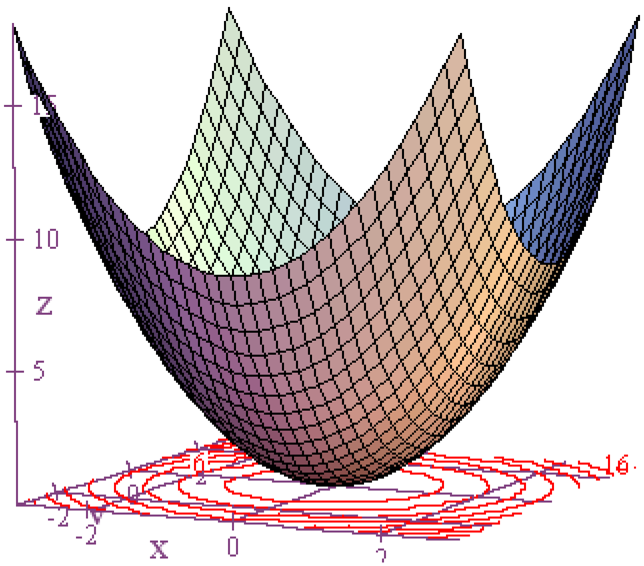
$$z = x^2 + y^2$$

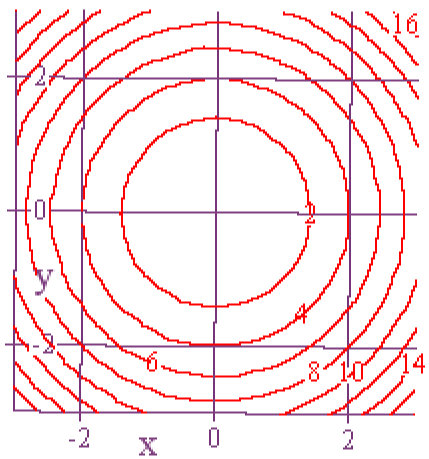
$$x^2 + y^2 = 4$$

$$z = 4$$



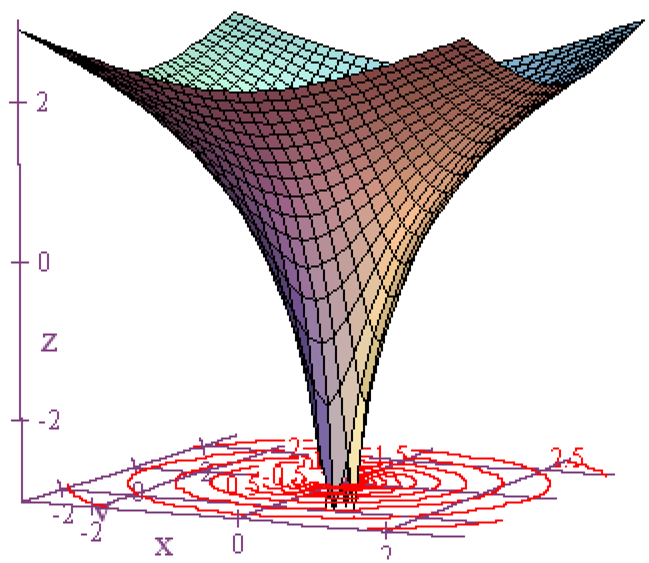
Small Hill

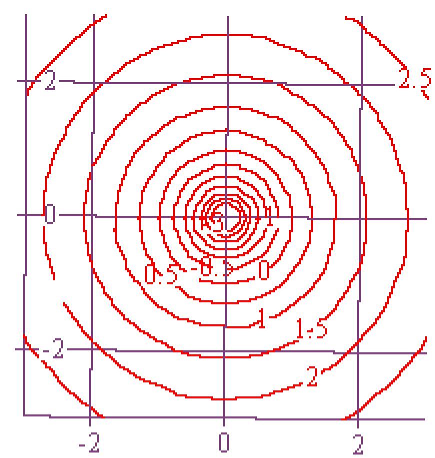




Notice, too, that anytime the variable expression is x^2+y^2, we're going to get level curves that are circles.

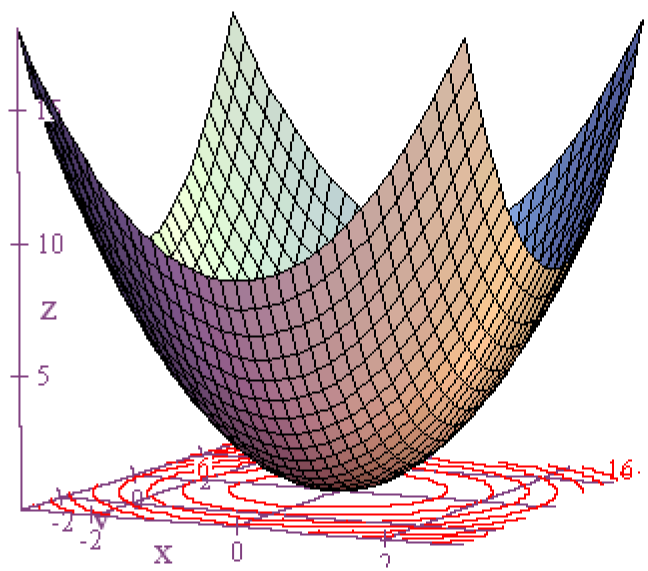
$$z = \ln\left(x^2 + y^2\right)$$

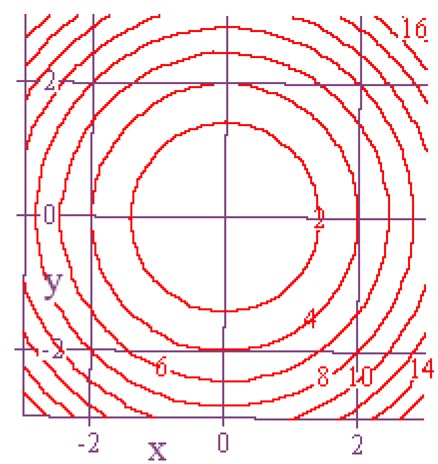




Here is the first of the three most important graphs to know. This one is called a *paraboloid.*

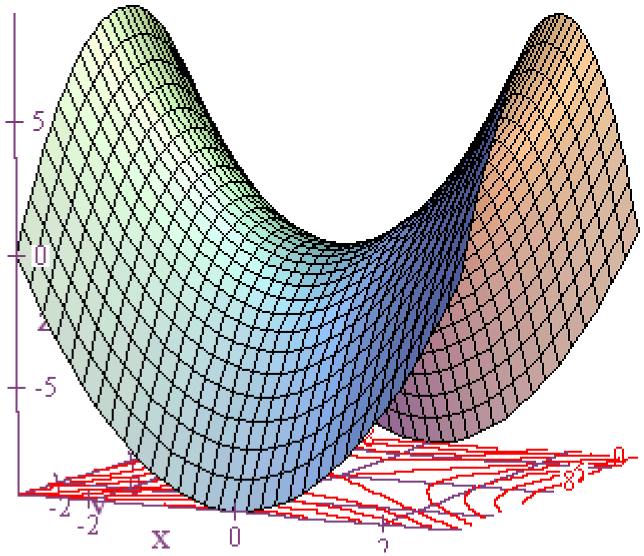
$$z = x^2 + y^2$$

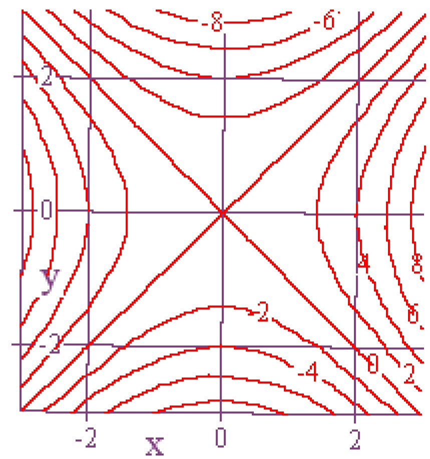




The next one is called a *saddle.*

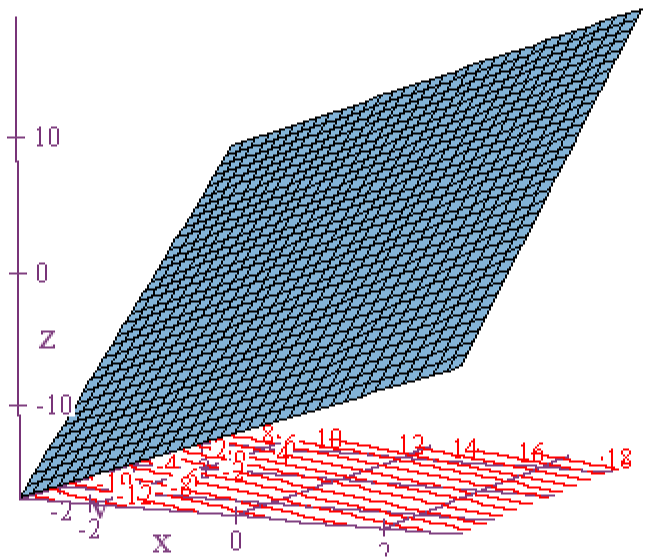
$$z = x^2 - y^2$$

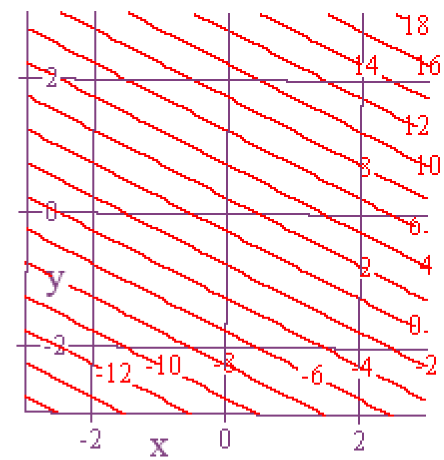




And finally, any function of the form z=ax+by+c results in the graph of a *plane.*

$$z = 2x + 4y + 1$$





Notice that the slope in the direction of positive *x* is 2, and the slope in the direction of positive *y* is 4. Also, the *z*-intercept is 1.

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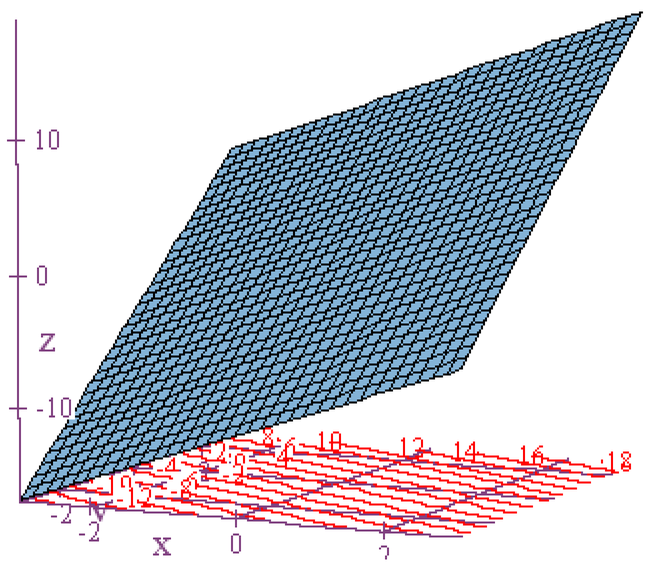
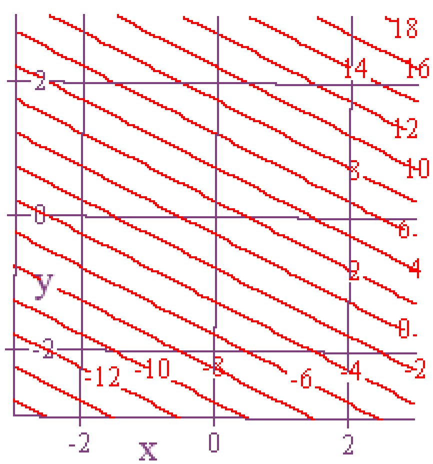
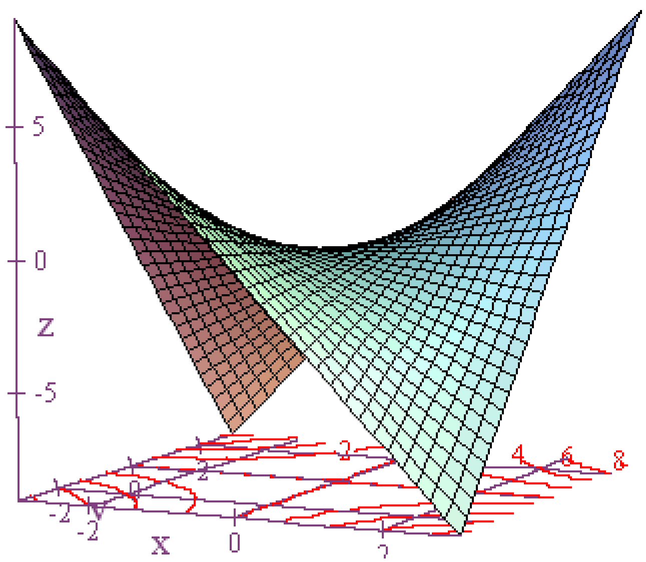
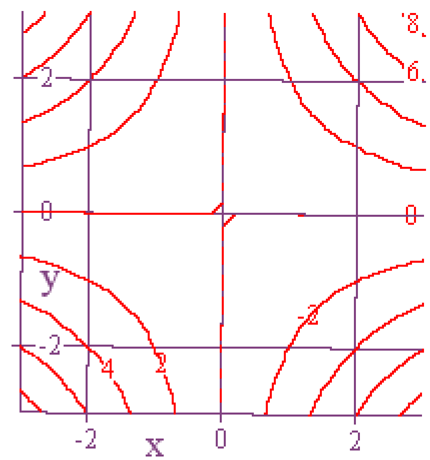




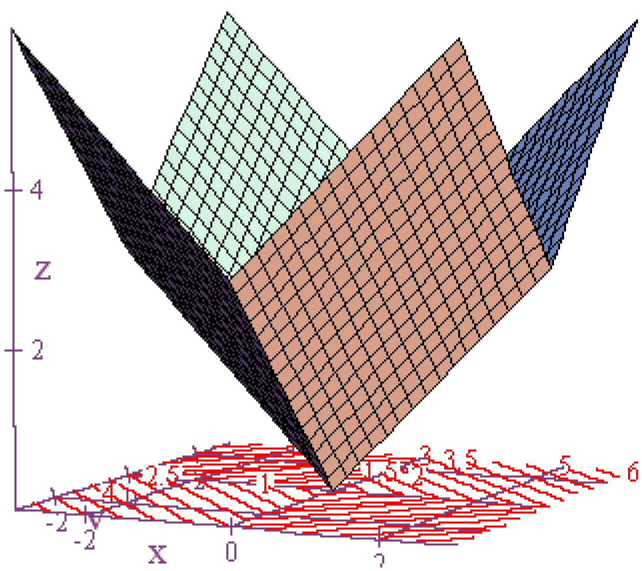
And lastly, here are a variety of interesting graphs for your viewing pleasure.

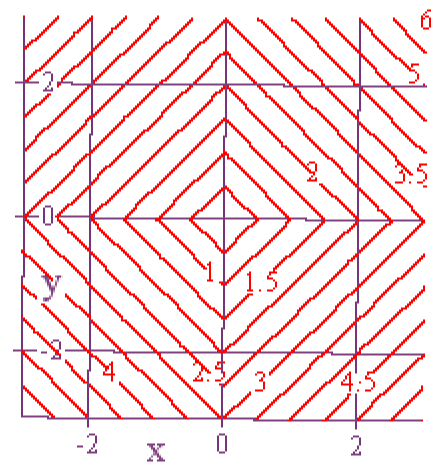
$z = xy$



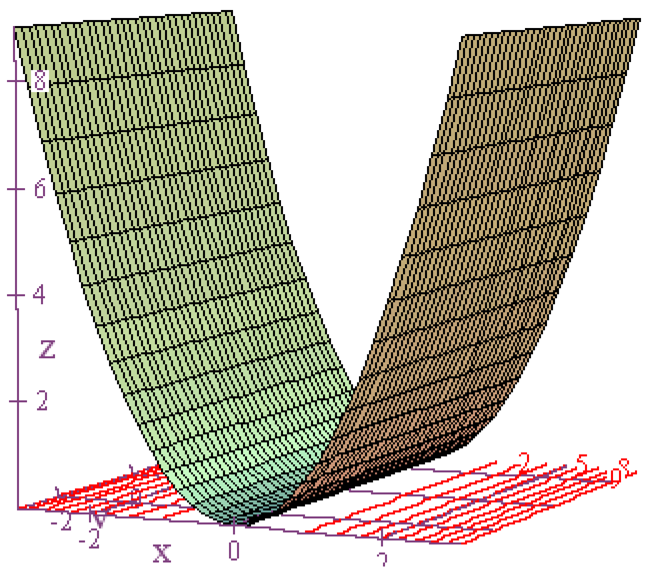


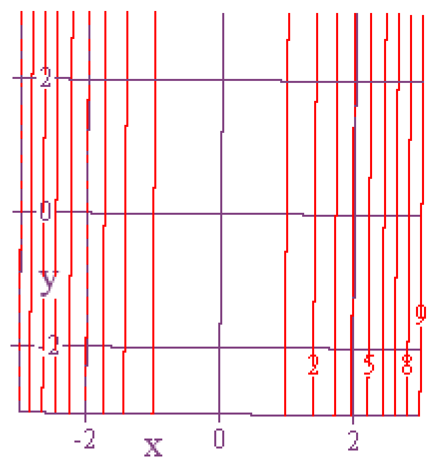
$$z = |x| + |y|$$



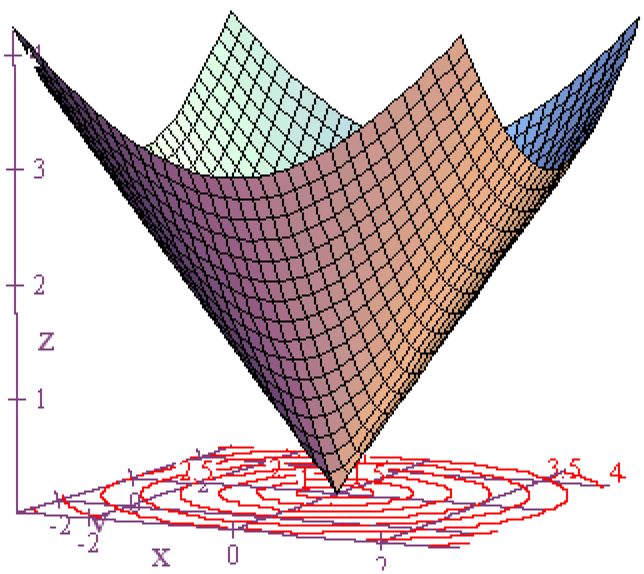


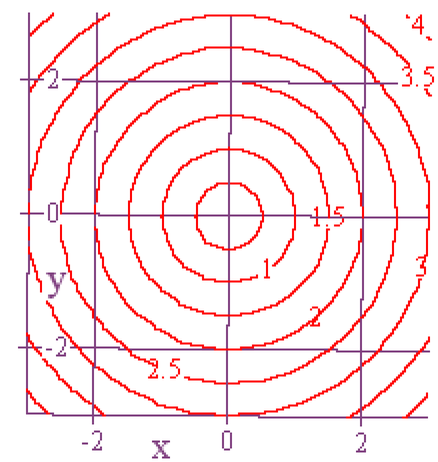
$$z = x^2$$



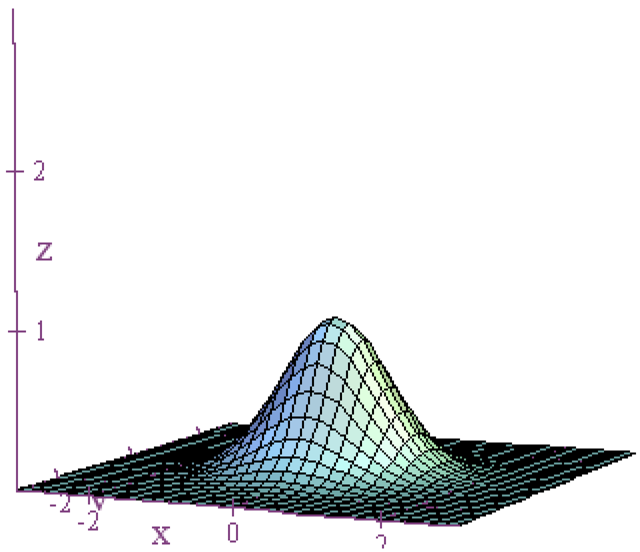


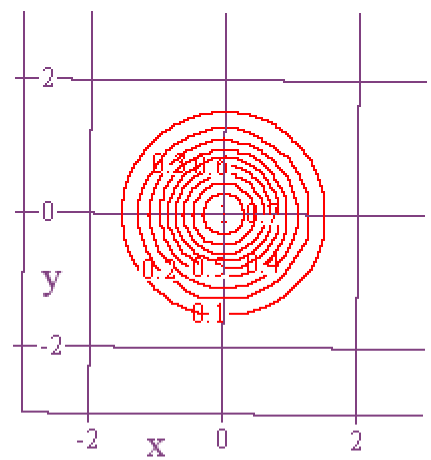
$$z = \sqrt{x^2 + y^2}$$



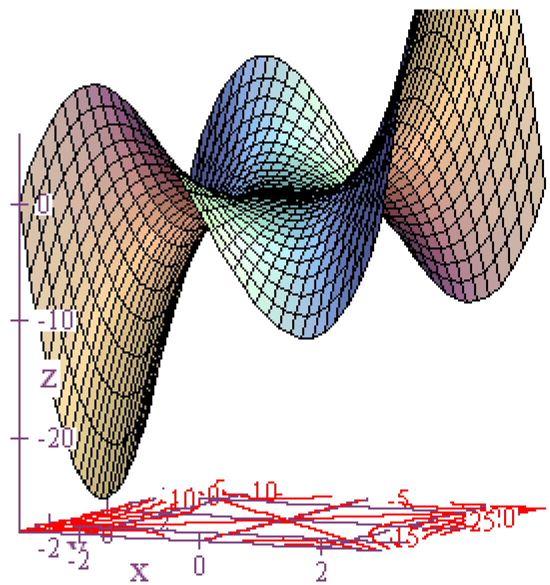


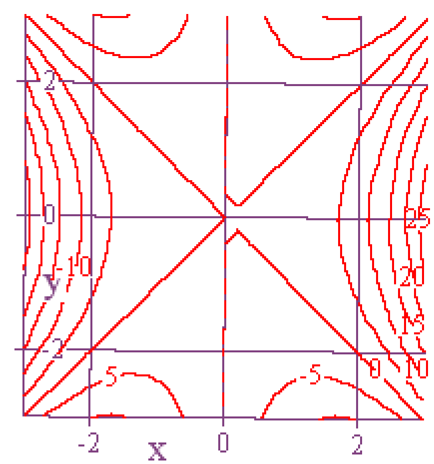
$$z = e^{-(x^2+y^2)}$$



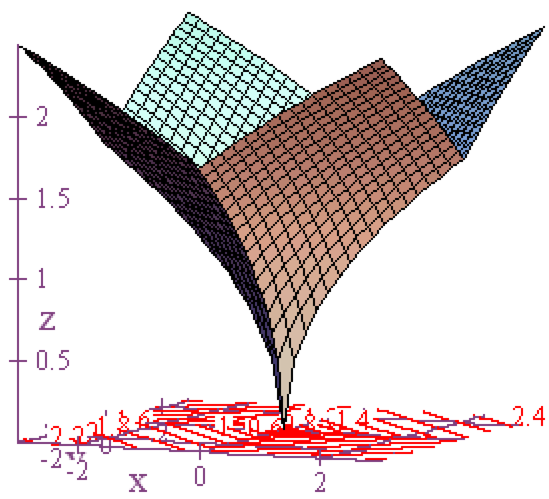


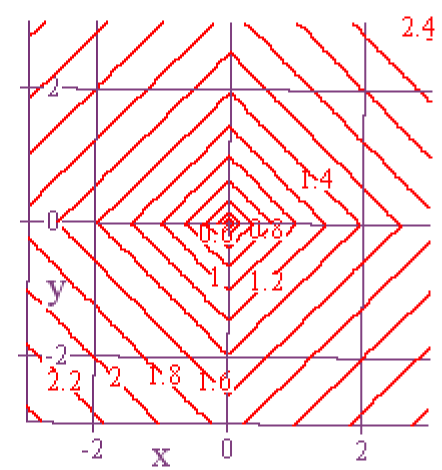
$$z = x^3 - xy^2$$



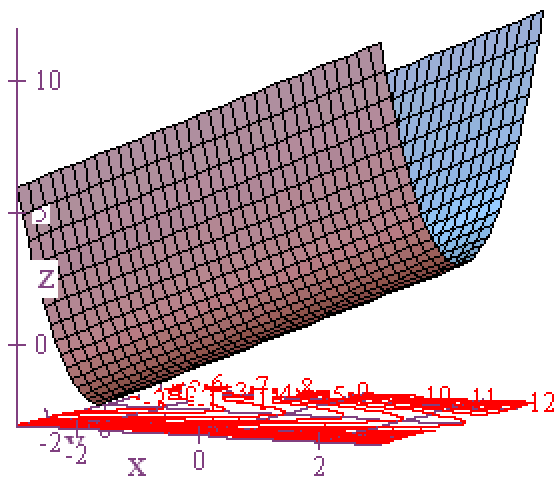


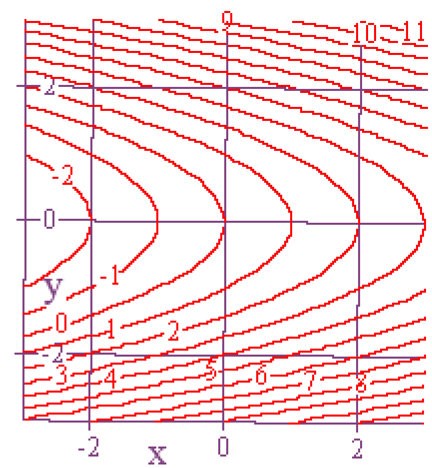
$$z = \sqrt{|x| + |y|}$$



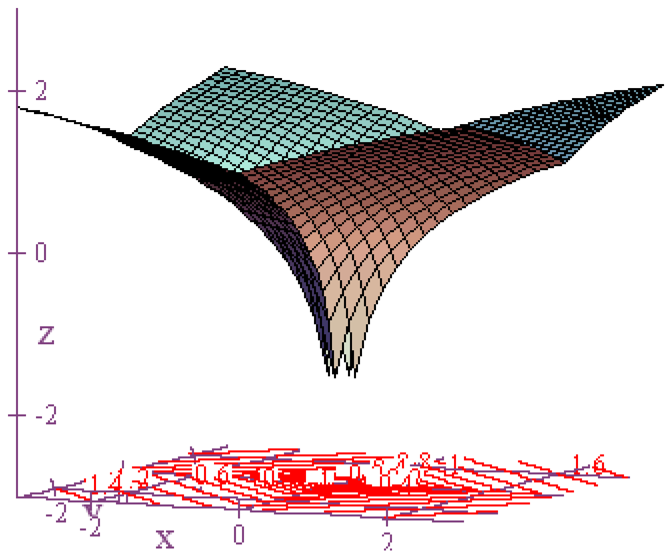


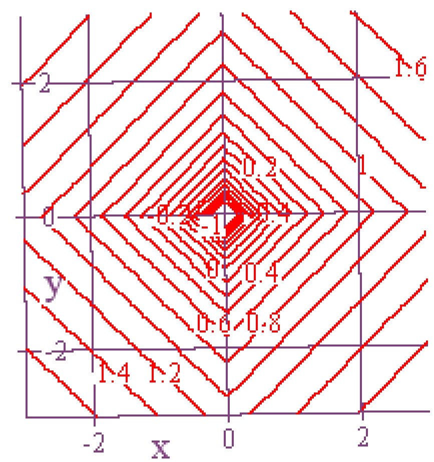
$$z = y^2 + x$$





$$z = \ln\left(|x| + |y|\right)$$





$$z = \frac{xy(x^2 - y^2)}{x^2 + y^2}$$