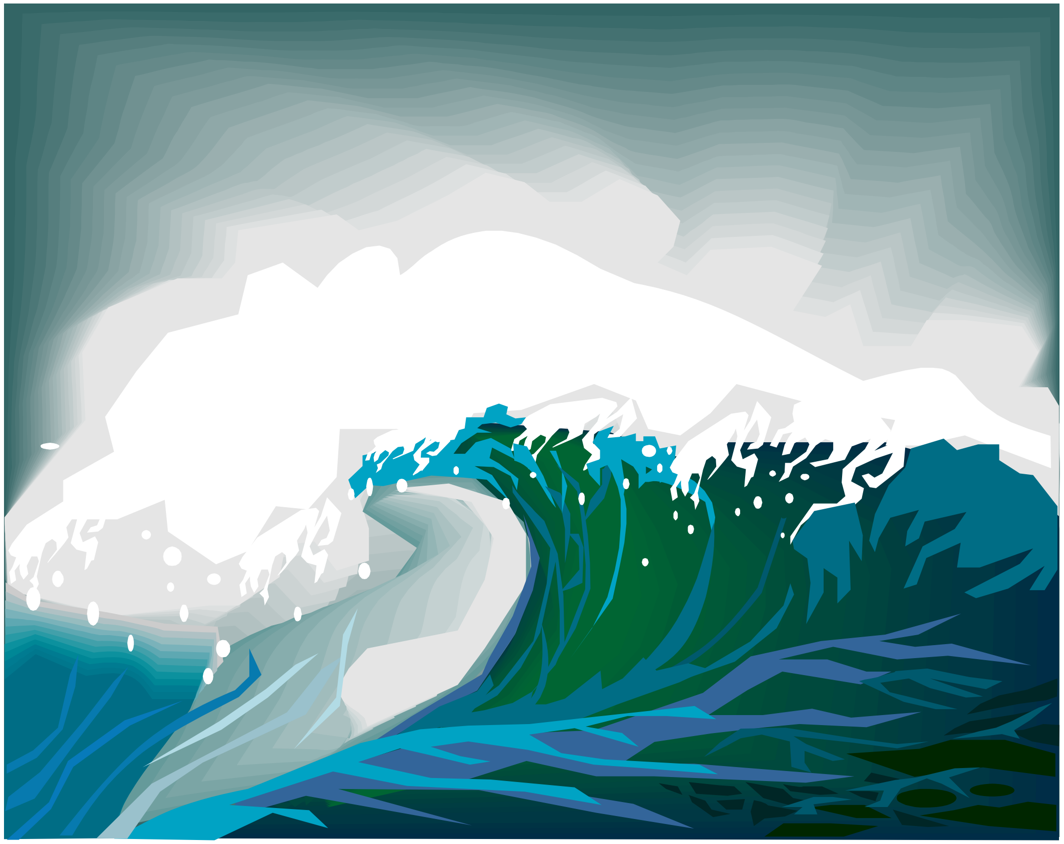
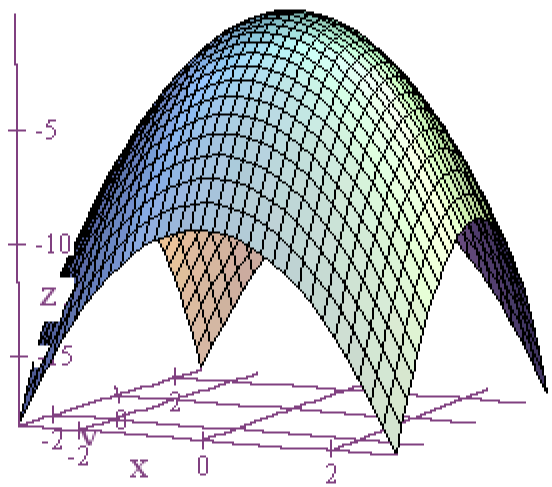
PARTIAL DERIVATIVES



Consider the function $z = f(x, y) = -x^2 - y^2$



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Thus, the partial derivative of $z = f(x, y) = -x^2 - y^2$ with respect to x is:

$$z_x = \frac{\partial f}{\partial x} = -2x$$

and the partial derivative of $z = f(x, y) = -x^2 - y^2$ with respect to y is:

$$z_y = \frac{\partial f}{\partial y} = -2y$$

If we consider the point $P = (2, -1, -5)$ on the surface of $z = f(x, y)$, then we can evaluate our partial derivatives at the *xy - coordinates* of this point. Thus,

$$z_x = \frac{\partial f}{\partial x} = -2x, \quad z_x(2, -1) = -2(2) = -4$$

$$z_y = \frac{\partial f}{\partial y} = -2y, \quad z_y(2, -1) = -2(-1) = 2$$

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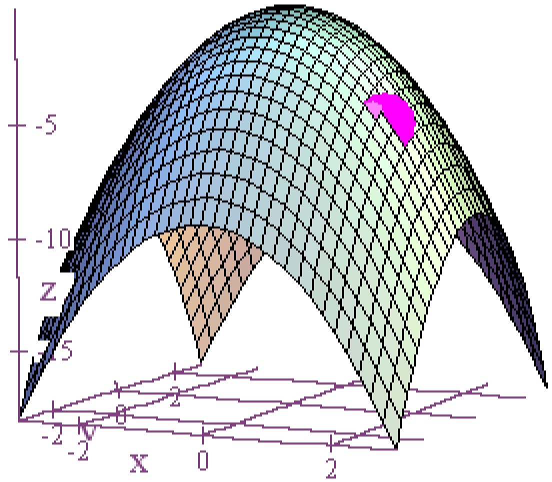
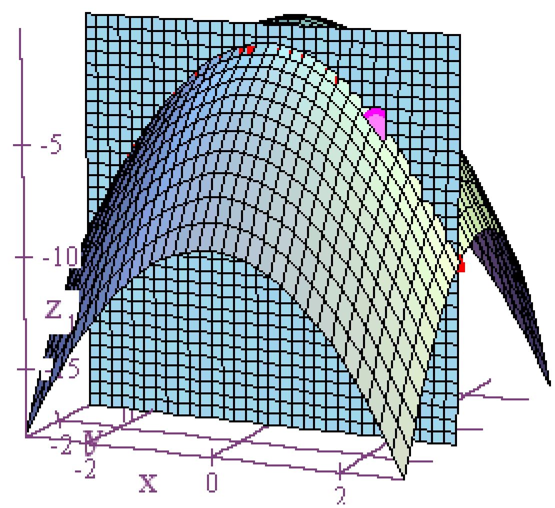
What does this mean?

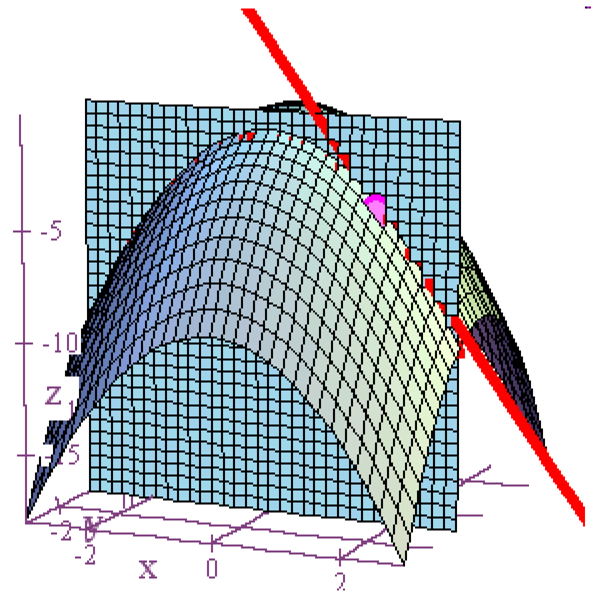
Let's look again at the graph of $z = f(x, y) = -x^2 - y^2$ with the point $P = (2, -1, -5)$ plotted on the surface.



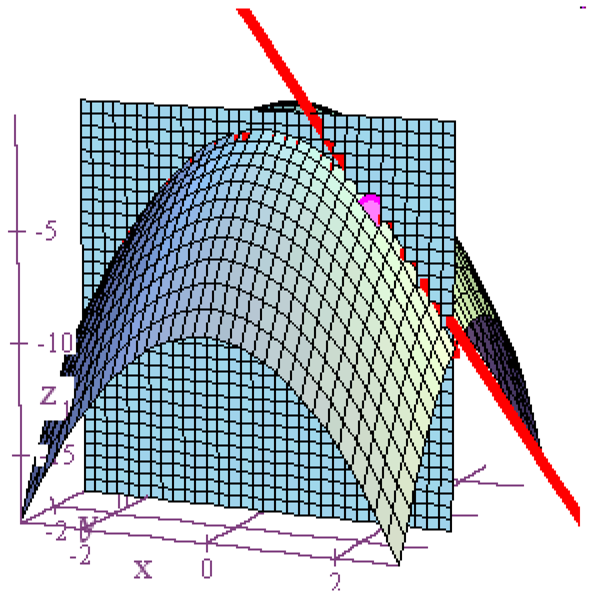
Slice through this surface with the plane $y = -1$, and we'll see a curve of intersection with the surface and the point $P = (2, -1, -5)$.



There is a line in the plane $y = -1$ that is tangent to the curve of intersection with the surface at the point $P = (2, -1, -5)$.

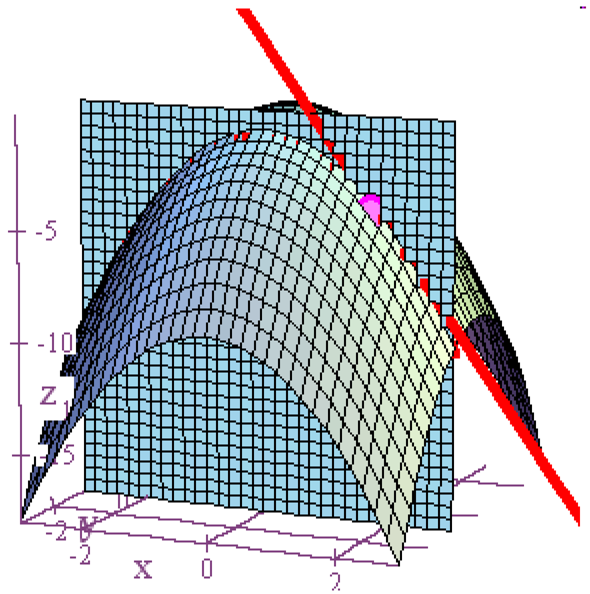


There is a line in the plane $y=-1$ that is tangent to the curve of intersection with the surface at the point $P=(2,-1,-5)$.



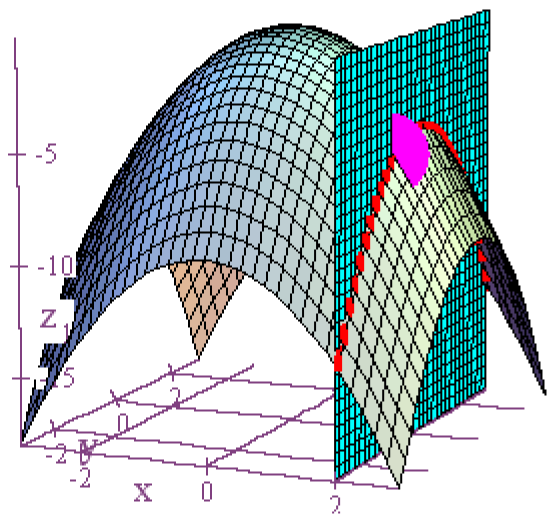
The slope of this line is $z_x(2,-1)=-2(2)=-4$.

This also means that if we are at the point $P = (2, -1, -5)$ on our surface and if we go 1 unit in the direction of positive x, then z will decrease by approximatly 4 units.

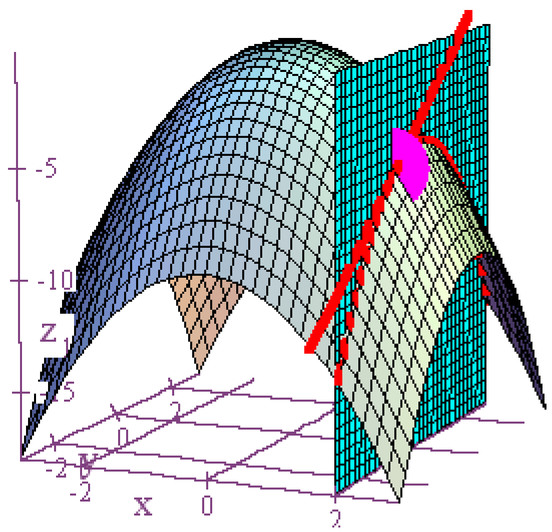


The slope of this line is $z_x(2, -1) = -2(2) = -4$.

Similarly, slice through the surface with the plane $x = 2$.



There is a line in the plane $x = 2$ that is tangent to the curve of intersection with the surface at the point $P = (2, -1, -5)$.



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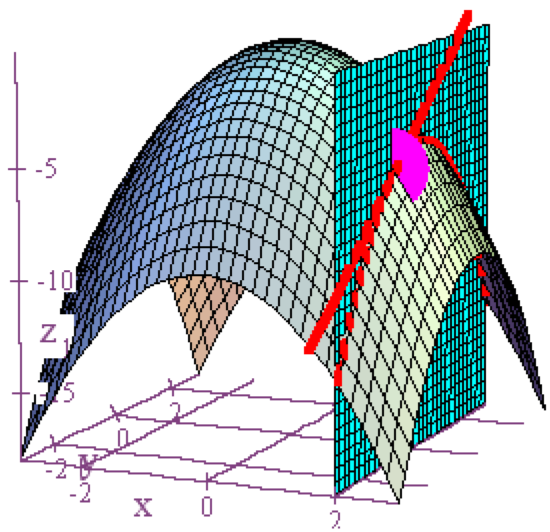
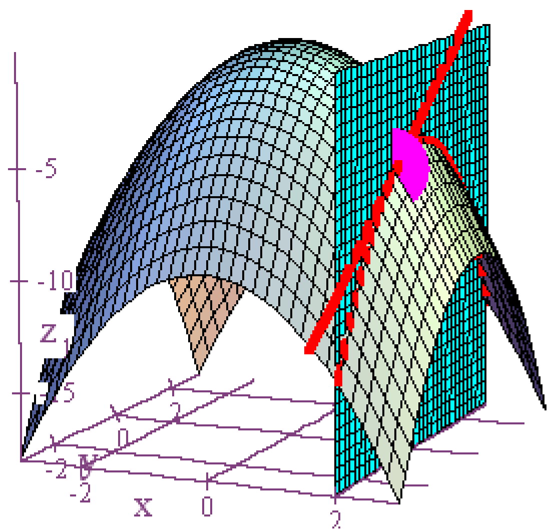


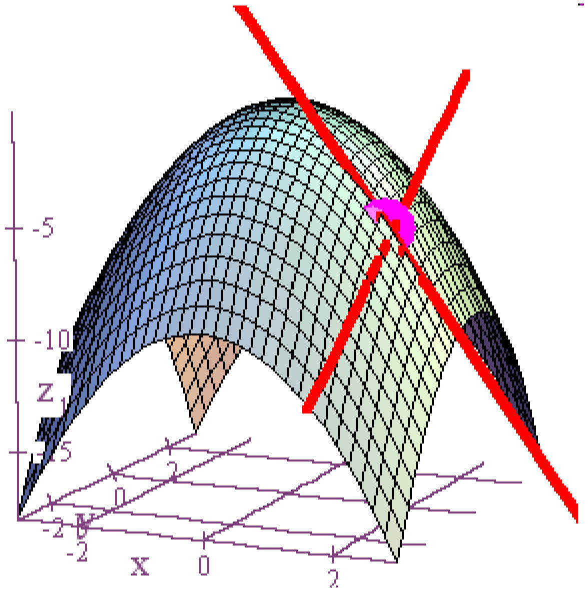
The slope of this line is $z_y(2, -1) = -2(-1) = 2$.

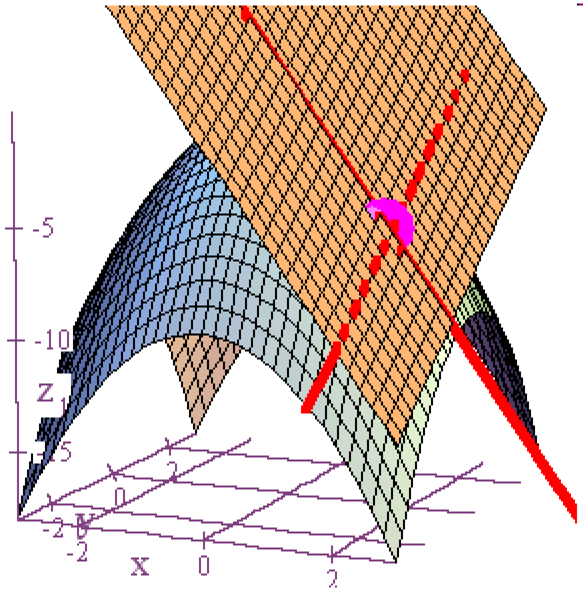
This also means that if we are at the point $P = (2, -1, -5)$ on our surface and if we go 1 unit in the direction of positive y, then z will increase by approximately 2 units.



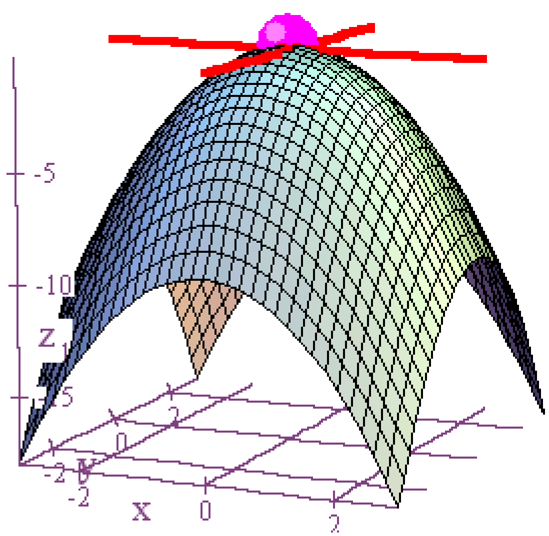
The slope of this line is $z_y(2, -1) = -2(-1) = 2$.

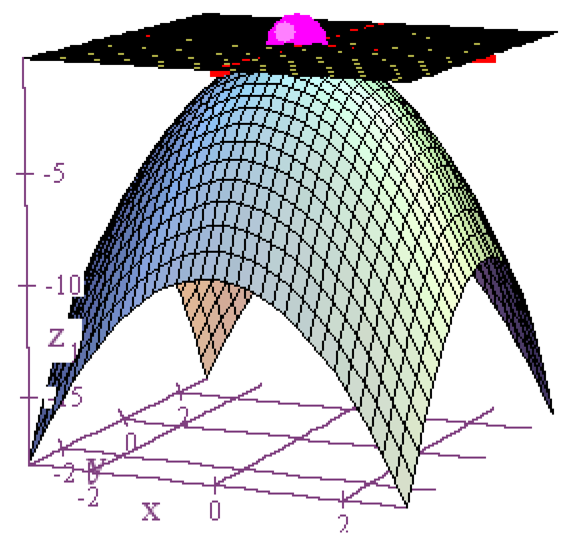
If we look at the two tangent lines together, we can see that they define a plane that is tangent to the surface at the point $P = (2, -1, -5)$.





Now notice that if we are at the maximum point on our surface, then the tangent plane will be horizontal and both our tangent lines will have slope zero.





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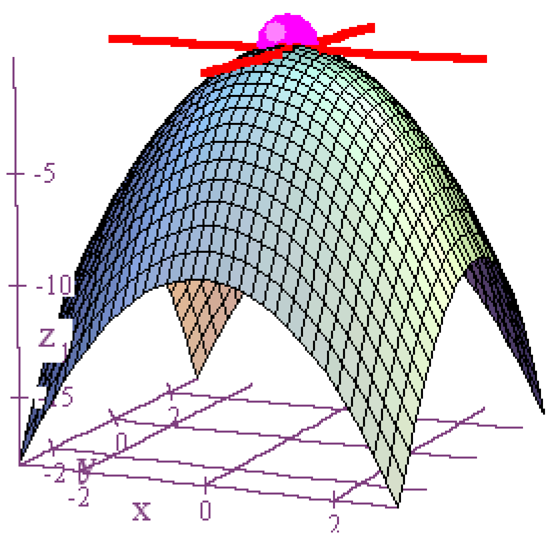

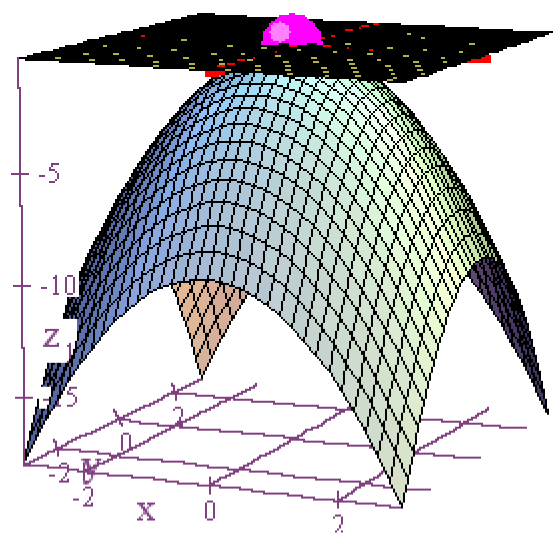

But that's another story for another day!