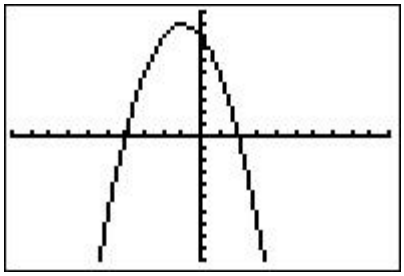
RELATIVE EXTREMA EXERCISES – ANSWERS

For each problem below, identify the critical point(s) and the coordinates of all relative extrema and label each as either a relative maximum or a relative minimum. Give exact answers in fraction form

1. $f(x) = -x^2 - 2x + 8$



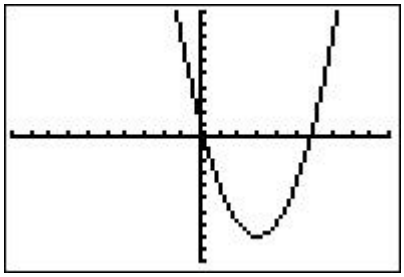
$f'(x) = -2x - 2 = -2(x+1)$

$f'(x) = 0 \Rightarrow -2(x+1) = 0 \Rightarrow x = -1$ critical point

$f(-1) = 9$ is a relative maximum

$(-1, 9)$ is a relative maximum point

2. $f(x) = x^2 - 6x + 1$



$f'(x) = 2x - 6 = 2(x-3)$

$f'(x) = 0 \Rightarrow 2(x-3) = 0 \Rightarrow x = 3$ critical point

$f(3) = -8$ is a relative minimum

$(3, -8)$ is a relative minimum point

3. $f(x) = x^4 - 5x^2 + 1$

$f'(x) = 4x^3 - 10x = 2x(2x^2 - 5)$

$f'(x) = 0 \Rightarrow x = 0$ or $2x^2 - 5 = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{\frac{5}{2}} = \frac{\pm\sqrt{10}}{2}$ critical points

$f(0) = 1$ is a relative maximum

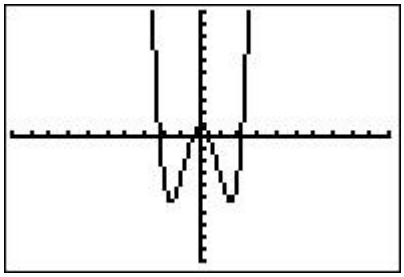
f($-\sqrt{5/2}$) $= -21/4$ is a relative minimum

f($\sqrt{5/2}$) $= -21/4$ is a relative minimum

(0,1) is a relative maximum point

($-\sqrt{5/2}, -21/4$) is a relative minimum point

($\sqrt{5/2}, -21/4$) is a relative minimum point



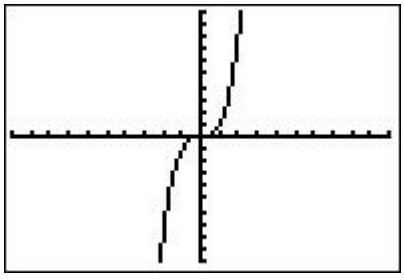
4. $f(x) = x^3$

$f'(x) = 3x^2$

$f'(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0$ critical point

$f(0) = 0$ is a neither a relative minimum nor a relative minimum

No relative extrema



5. $f(x) = x^3 - x^2 - x - 1$

$f'(x) = 3x^2 - 2x - 1 = (3x+1)(x-1)$

$f'(x) = 0 \Rightarrow 3x+1 = 0$ or $x-1 = 0 \Rightarrow x = -1/3$ or $x = 1$ critical points

$f(-1/3) = -22/27$ is a relative maximum

$f(1) = -2$ is a relative minimum

$(-1/3, -22/27)$ is a relative maximum point

$(1, -2)$ is a relative minimum point