2ND ORDER PARTIAL DERIVATIVES

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Theorem: If z_{xy} and z_{yx} are continuous at (a,b), an interior point of the domain, then $z_{xy}(a,b) = z_{yx}(a,b)$.

What do the 2nd order partials tell us?

$$z = x^2 - y^2$$

$$z_x = 2x$$

$$z_y = -2y$$

$$z_{xx} = 2$$

$$z_{yy} = -2$$

In this case, they tell us that for a fixed value of *y*, the curve of intersection will be concave up.

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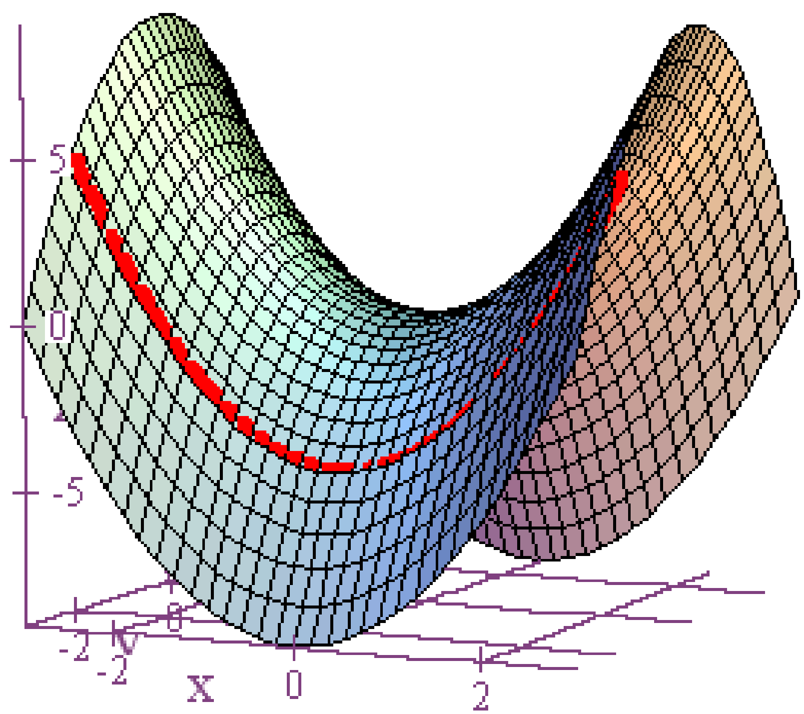

And for a fixed value of x,

the curve of intersection will be concave down.

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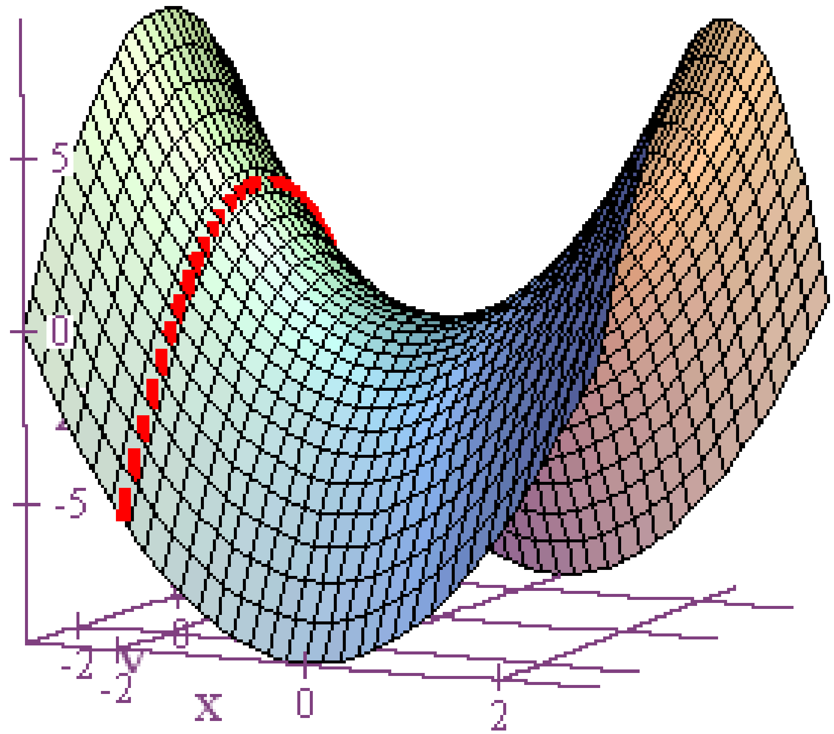

The geometric interpretations of the mixed partials are harder to visualize. However, the good news is that we're not going have to worry about them for what we want to accomplish.

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