DIVERGENCE THEOREM IN THREE DIMENSIONS

In each problem below you are given a volume V bounded by a surface S along with a vector field F. If N is an outward pointing unit normal vector, then the outward flux across the surface created by the vector field F is, by the Divergence (Gauss') Theorem, equal to $\iint_S \vec{F} \bullet N dS = \iiint_V div\,\vec{F}\, dV = \iiint_V \nabla \bullet F\, dV$. Evaluate this integral for each problem below

1. V is the solid ball with surface S defned by $x^2 + y^2 + z^2 = 1$.
   $F = x\hat{i} + y\,\hat{j} + z\,\hat{k}$

2. V is the cube defned by $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$, and S is the surface of the cube.
   $F = x^2\,\hat{i} + y\,\hat{j} + z\,\hat{k}$

3. V is the cylinder defned by $-1 \le x \le 1, -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}, 0 \le z \le 4$,
   (bounded below by $z = 0$ and above by $z = 4$), and S is the surface of the cylinder.
   $F = y\hat{i} - x\,\hat{j}$

4. V is the solid bounded by the *xy*-plane and the hemisphere $z = \sqrt{4 - x^2 - y^2}$,
   and S is the surface of V.
   $F = x^3\,\hat{i} + y^3\,\hat{j} + z^3\,\hat{k}$

5. V is the solid bounded by the portion in the first octant of the
   cylinder (including top and bottom) defined by $x^2 + y^2 = 1$ as z varies from 0 to 1,
   and S is the surface of V.
   $F = x^2\,\hat{i}$