STOKES' THEOREM IN THREE DIMENSIONS

In each problem below you are given a surface *S*, defined by $z = f(x,y)$, over a region *R*, defined by the given limits on *x* and *y*. Let C_R be the boundary of the region *R*, oriented counterclockwise, and let *C* be the corresponding bounding curve on the surface *S*, also oriented counterclockwise (Except for problem 4. On problem 4, let your bounding curves be oriented clockwise.). Then if *F* is a vector field and *N* is the upward pointing unit normal vector for the surface *S*, use the higher dimensional version of Stokes' Theorem, $\int_C \vec{F} \bullet d\vec{r} = \iint_S \left(curl\, F \bullet N \right) dS$, to measure the circulation around the curve *C* that is caused by the vector field *F*.

1. $S: z = -x^2 - y^2 + 4$
   $R: 0 \le x \le 1,\ 0 \le y \le 2$
   $F = z\hat{i} + x\hat{j} + y\hat{k}$

2. $S: z = y$
   $R: 0 \le x \le 1,\ 0 \le y \le 1 - x$
   $F = -3y^2\hat{i} + 4z\hat{j} + 6x\hat{k}$

3. $S: z = x^2 - y^2$
   $R: -1 \le x \le 1,\ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$
   $F = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

4. $S: z = 1 - x^2 - y^2$
   $R: -1 \le x \le 1,\ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$
   $F = z\hat{i} + x\hat{j} + y\hat{k}$
   (NOTE: On this problem, let C_R and *C* be oriented clockwise. This means that your unit normal *N* will be pointing downward instead of upward.)

5. $S: z = \frac{1}{2}\sqrt{1 - x^2 - y^2}$
   $R: -1 \le x \le 1,\ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$
   $F = x^2\hat{i} + y^2\hat{j} + z^2 \tan xy\,\hat{k}$
   (HINT: Use a simpler surface with the same bounding curve.)

6. Let $F = (-6y^2 + 6y)\hat{i} + (x^2 - 3z^2)\hat{j} - x^2\hat{k}$, and use Stokes' Theorem to show that the work done by *F* along any simple closed curve contained in the plane $x + 2y + z = 1$ is equal to zero.