AREA OF A PARALLELOGRAM - ANSWERS

Find the area of the parallelogram determined by the following vectors.

1. $\vec{u} = 2\hat{i} + 3\hat{j}$ and $\vec{v} = 3\hat{i} - 2\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 3 & -2 & 0 \end{vmatrix} = (-4-9)\hat{k} = -13\hat{k}$$

$$\|\vec{u} \times \vec{v}\| = 13$$

2. $\vec{u} = 2\hat{i} + 3\hat{j}$ and $\vec{v} = 4\hat{i} + 6\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 4 & 6 & 0 \end{vmatrix} = 0\hat{k} = \vec{0}$$

$$\|\vec{u} \times \vec{v}\| = 0$$

Zero area, the vectors are parallel

3. $\vec{u} = 2\hat{i} + 3\hat{j}$ and $\vec{v} = -6\hat{i} - 9\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ -6 & -9 & 0 \end{vmatrix} = (-18+18)\hat{k} = 0\hat{k} = \vec{0}$$

$$\|\vec{u} \times \vec{v}\| = 0$$

Zero area, the vectors are parallel

4. $\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 3\hat{i} - 2\hat{j} + \hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & -2 & 1 \end{vmatrix} = (3+2)\hat{i} - (2-3)\hat{j} + (-4-9)\hat{k} = 5\hat{i} + \hat{j} - 13\hat{k}$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{25+1+169} = \sqrt{195} \approx 13.96$$

5. $\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{v} = 2\hat{i} + 2\hat{j} - 10\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 2 & 2 & -10 \end{vmatrix} = (-30 - 2)\hat{i} - (-20 - 2)\hat{j} + (4 - 6)\hat{k} = -32\hat{i} + 22\hat{j} - 2\hat{k}$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{1024 + 484 + 4} = \sqrt{1512} = 6\sqrt{42} \approx 38.88$$

6. $\vec{u} = -\hat{i} - \hat{j} - 5\hat{k}$ and $\vec{v} = 2\hat{i} + 2\hat{j} - 10\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & -5 \\ 2 & 2 & -10 \end{vmatrix} = (10 + 10)\hat{i} - (10 + 10)\hat{j} + (-2 + 2)\hat{k} = 20\hat{i} + 20\hat{j} + 0\hat{k}$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{400 + 400 + 0} = \sqrt{800} = 20\sqrt{2} \approx 28.28$$