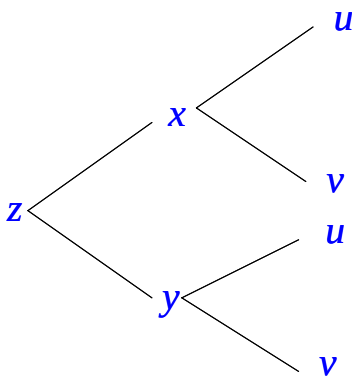
CHAIN RULE DIAGRAMS – ANSWERS

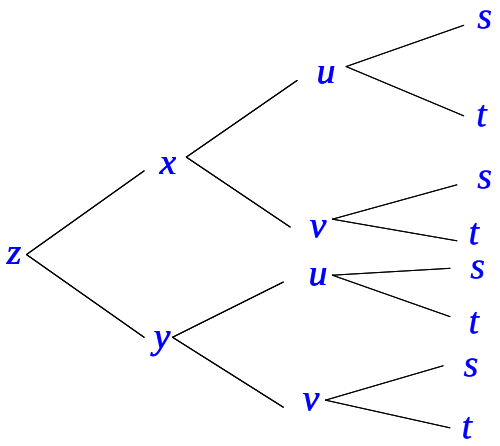
Construct a tree diagram and a chain rule formula for each of the indicated derivatives.

1. $z = f(x,y),\ x = x(u,v),\ y = y(u,v),\ \dfrac{\partial z}{\partial u} = ?$



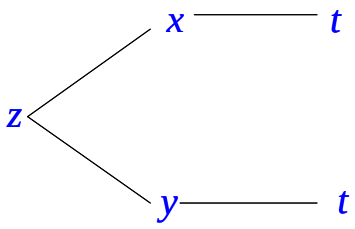
$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

2. $z = f(x,y),\ x = x(u,v),\ y = y(u,v),\ u = u(s,t),\ v = v(s,t),\ \dfrac{\partial z}{\partial s} = ?$



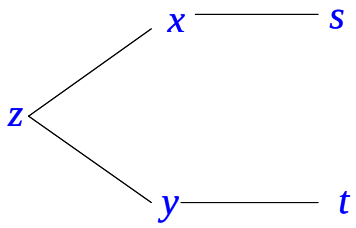
$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u}\frac{\partial u}{\partial s} + \frac{\partial z}{\partial x}\frac{\partial x}{\partial v}\frac{\partial v}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}\frac{\partial u}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}\frac{\partial v}{\partial s}$$

3. $z = f(x, y),\ x = x(t),\ y = y(t),\ \dfrac{dz}{dt} = ?$



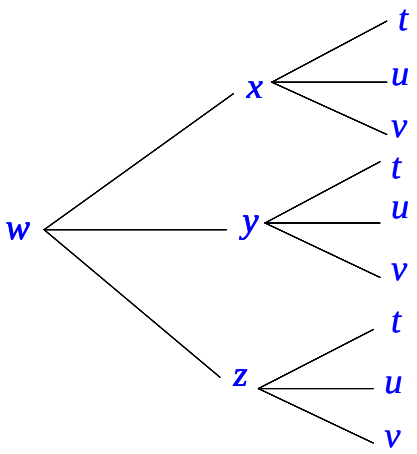
$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

4. $z = f(x, y),\ x = x(s),\ y = y(t),\ \dfrac{\partial z}{\partial t} = ?$



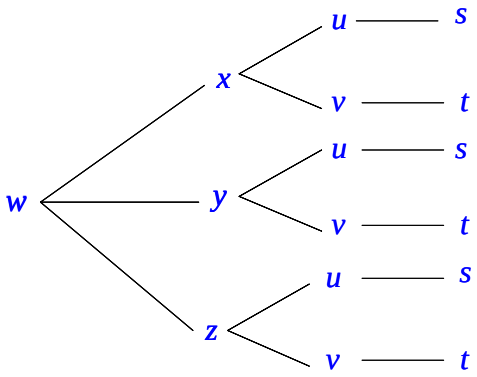
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial y}\frac{dy}{dt}$$

5. $w = f(x,y,z),\ x = x(t,u,v),\ y = y(t,u,v),\ z = z(t,u,v),\ \dfrac{\partial w}{\partial t} = ?$



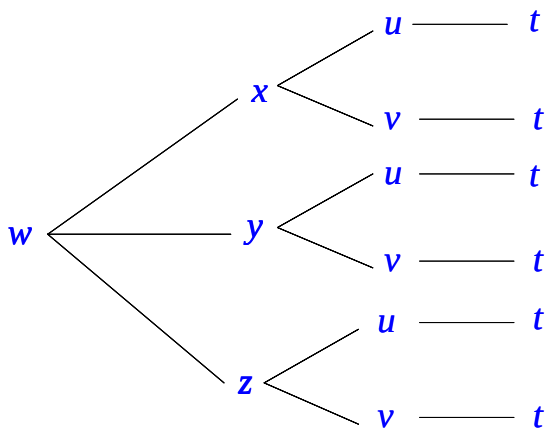
$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

6. $w = f(x,y,z),\ x = x(u,v),\ y = y(u,v),\ z = z(u,v),\ u = u(s),\ v = v(t),\ \dfrac{\partial w}{\partial t} = ?$



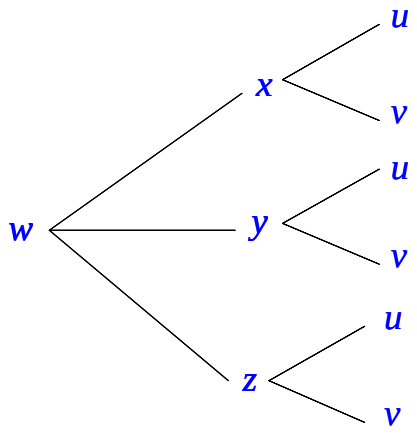
$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v}\frac{dv}{dt} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v}\frac{dv}{dt} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}\frac{dv}{dt}$$

7. $w = f(x,y,z),\ x = x(u,v),\ y = y(u,v),\ z = z(u,v),\ u = u(t),\ v = v(t),\ \dfrac{dw}{dt} = ?$



$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u}\frac{du}{dt} + \frac{\partial w}{\partial x}\frac{\partial x}{\partial v}\frac{dv}{dt} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u}\frac{du}{dt} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v}\frac{dv}{dt} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial u}\frac{du}{dt} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}\frac{dv}{dt}$$

8. $w = f(x,y,z),\ x = x(u,v),\ y = y(u,v),\ z = z(u,v),\ \dfrac{\partial w}{\partial v} = ?$



$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial v}$$