COMPONENTS AND PROJECTIONS - ANSWERS

(1-5) In each of the problems below, you are given a force vector $\vec{F}$ and a distance vector $\vec{d}$. Suppose the magnitude of $\vec{F}$ corresponds to the number of pounds of force and the magnitude of $\vec{d}$ corresponds to a distance in feet that an object is moved by the force. For each of the problems below find $comp_{\vec{d}}\vec{F}$, $proj_{\vec{d}}\vec{F}$, and the work done by $\vec{F}$ in moving the object the length of $\vec{d}$. Give exact answers, and on the latter, use units of *foot-pounds*.

1. $\vec{F} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{d} = 2\hat{i} + 2\hat{j} + 5\hat{k}$
   work $= 21$ foot-pounds
   $$comp_{\vec{d}}\vec{F} = \frac{21}{\sqrt{33}} = \frac{7\sqrt{33}}{11}$$
   $$proj_{\vec{d}}\vec{F} = \frac{21}{33}\vec{d} = \frac{7}{11}\vec{d} = \frac{14}{11}\hat{i} + \frac{14}{11}\hat{j} + \frac{35}{11}\hat{k}$$

2. $\vec{F} = 3\hat{i} + \hat{j} + 4\hat{k}$, $\vec{d} = 8\hat{i} + 2\hat{j} + 6\hat{k}$
   work $= 50$ foot-pounds
   $$comp_{\vec{d}}\vec{F} = \frac{25}{\sqrt{26}} = \frac{25\sqrt{26}}{26}$$
   $$proj_{\vec{d}}\vec{F} = \frac{25}{52}\vec{d} = \frac{50}{13}\hat{i} + \frac{25}{26}\hat{j} + \frac{75}{26}\hat{k}$$

3. $\vec{F} = 3\hat{i} + 2\hat{j}$, $\vec{d} = 10\hat{i}$
   work $= 30$ foot-pounds
   $$comp_{\vec{d}}\vec{F} = \frac{30}{10} = 3$$
   $$proj_{\vec{d}}\vec{F} = \frac{30}{100}\vec{d} = 3\hat{i}$$

4. $\vec{F} = \hat{i} + \hat{j}$, $\vec{d} = 5\hat{i} + \hat{j}$
   work $= 6$ foot-pounds
   $$comp_{\vec{d}}\vec{F} = \frac{6}{\sqrt{26}} = \frac{3\sqrt{26}}{13}$$
   $$proj_{\vec{d}}\vec{F} = \frac{6}{26}\vec{d} = \frac{3}{13}\vec{d} = \frac{15}{13}\hat{i} + \frac{3}{13}\hat{j}$$

5. $\vec{F} = 2\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{d} = 2\hat{i} + 2\hat{j} + 2\hat{k}$
   work $= 12$ foot-pounds
   $$comp_{\vec{d}}\vec{F} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$
   $$proj_{\vec{d}}\vec{F} = \left(\frac{12}{12}\right)\vec{d} = 2\hat{i} + 2\hat{j} + 2\hat{k}$$

6. Find the component of $\vec{v} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ in the direction of the unit vector (a) $\hat{i}$, (b) $\hat{j}$, (c) $\hat{k}$, and (d) $\vec{u} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$.

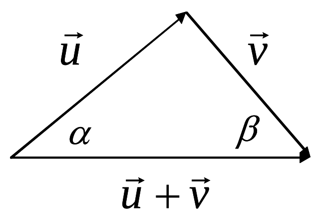
   (a) $comp_{\hat{i}}\vec{v} = \vec{v} \bullet \hat{i} = 4$

   (b) $comp_{\hat{j}}\vec{v} = \vec{v} \bullet \hat{j} = 5$

   (c) $comp_{\hat{k}}\vec{v} = \vec{v} \bullet \hat{k} = 6$

   (d) $comp_{\vec{u}}\vec{v} = \vec{v} \bullet \vec{u} = \dfrac{4\sqrt{3}+5}{2}$

7. Explain why the triangle inequality, $\|\vec{u}+\vec{v}\| \le \|\vec{u}\| + \|\vec{v}\|$, is true for the diagram below. (NOTE: The triangle inequality is also true for all real numbers *a* and *b*.)



Clearly, $\|\vec{u}+\vec{v}\| = comp_{\vec{u}+\vec{v}}\vec{u} + comp_{\vec{u}+\vec{v}}\vec{v} = \|\vec{u}\|\cos\alpha + \|\vec{v}\|\cos\beta$. Furthermore, since $\|\vec{u}\|\cos\alpha \le \|\vec{u}\|$ and $\|\vec{v}\|\cos\beta \le \|\vec{v}\|$, it follows that $\|\vec{u}+\vec{v}\| = \|\vec{u}\|\cos\alpha + \|\vec{v}\|\cos\beta \le \|\vec{u}\| + \|\vec{v}\|$.