COMPONENTS OF ACCELERATION – ANSWERS

For each of the following curves, find formulas for the tangential and normal components of acceleration.

1. $\vec{r}(t) = 4\cos t\,\hat{i} + 4\sin t\,\hat{j}$

$$\vec{v}(t) = -4\sin t\,\hat{i} + 4\cos t\,\hat{j}$$
$$\|\vec{v}(t)\| = \sqrt{16\sin^2 t + 16\cos^2 t} = \sqrt{16} = 4$$
$$T = \frac{\vec{v}}{\|\vec{v}\|} = -\sin t\,\hat{i} + \cos t\,\hat{j}$$
$$\vec{a}(t) = -4\cos t\,\hat{i} - 4\sin t\,\hat{j}$$
$$\|\vec{a}(t)\| = \sqrt{16\cos^2 t + 16\sin^2 t} = \sqrt{16} = 4$$

$$\vec{a}_T = \|\vec{v}(t)\|' = (4)' = 0$$
$$\vec{a}_T = \vec{a} \bullet T = 4\sin t\cos t - 4\sin t\cos t = 0$$
$$\vec{a}_N = \sqrt{\|\vec{a}(t)\|^2 - \left(\|\vec{v}(t)\|'\right)^2} = \sqrt{16 - 0} = 4$$

2. $\vec{r}(t) = (2 + 2t)\,\hat{i} + (1 + 3t)\,\hat{j}$

$$\vec{v}(t) = 2\,\hat{i} + 3\,\hat{j}$$
$$\|\vec{v}(t)\| = \sqrt{4 + 9} = \sqrt{13}$$
$$T = \frac{\vec{v}}{\|\vec{v}\|} = \frac{2}{\sqrt{13}}\,\hat{i} + \frac{3}{\sqrt{3}}\,\hat{j}$$
$$\vec{a}(t) = 0\,\hat{i} + 0\,\hat{j} = \vec{0}$$
$$\|\vec{a}(t)\| = \sqrt{0^2 + 0^2} = \sqrt{0} = 0$$

$$\vec{a}_T = \|\vec{v}(t)\|' = (\sqrt{13})' = 0$$
$$\vec{a}_T = \vec{a} \bullet T = 0 + 0 = 0$$
$$\vec{a}_N = \sqrt{\|\vec{a}(t)\|^2 - \left(\|\vec{v}(t)\|'\right)^2} = \sqrt{0^2 - 0^2} = \sqrt{0} = 0$$

3. $\vec{r}(t) = 2t\,\hat{i} + t^2\,\hat{j} + \frac{t^3}{3}\hat{k}$

$$\vec{v}(t) = 2\hat{i} + 2t\,\hat{j} + t^2\,\hat{k}$$
$$\|\vec{v}(t)\| = \sqrt{4+4t^2+t^4} = \sqrt{(t^2+2)^2} = t^2+2$$
$$T = \frac{\vec{v}}{\|\vec{v}\|} = \frac{2}{t^2+2}\hat{i} + \frac{2t}{t^2+2}\hat{j} + \frac{t^2}{t^2+2}\hat{k}$$
$$\vec{a}(t) = 2\,\hat{j} + 2t\,\hat{k}$$
$$\|\vec{a}(t)\| = \sqrt{4+4t^2}$$

$$a_T = \|\vec{v}(t)\|' = 2t$$
$$\vec{a}_T = \vec{a}\bullet T = \frac{4t+2t^3}{t^2+2} = \frac{2t(2+t^2)}{t^2+2} = 2t$$
$$a_N = \sqrt{\|\vec{a}(t)\|^2 - \left(\|\vec{v}(t)\|'\right)^2} = \sqrt{4+4t^2-4t^2} = \sqrt{4} = 2$$

4. $\vec{r}(t) = e^t\,\hat{i} + e^{-t}\,\hat{j} + t\sqrt{2}\,\hat{k}$

$$\vec{v}(t) = e^t\,\hat{i} - e^{-t}\,\hat{j} + \sqrt{2}\,\hat{k}$$
$$\|\vec{v}(t)\| = \sqrt{(e^t)^2 + (e^{-t})^2 + 2} = \sqrt{(e^t+e^{-t})^2} = e^t + e^{-t}$$
$$T = \frac{\vec{v}}{\|\vec{v}\|} = \frac{e^t}{e^t+e^{-t}}\hat{i} - \frac{e^{-t}}{e^t+e^{-t}}\hat{j} + \frac{\sqrt{2}}{e^t+e^{-t}}\hat{k}$$
$$\vec{a}(t) = e^t\,\hat{i} + e^{-t}\,\hat{j}$$
$$\|\vec{a}(t)\| = \sqrt{e^{2t}+e^{-2t}}$$

$$a_T = \|\vec{v}(t)\|' = e^t - e^{-t}$$
$$\vec{a}_T = \vec{a}\bullet T = \frac{(e^t)^2}{e^t+e^{-t}} - \frac{(e^{-t})^2}{e^t+e^{-t}} = \frac{(e^t+e^{-t})(e^t-e^{-t})}{e^t+e^{-t}} = e^t - e^{-t}$$
$$a_N = \sqrt{\|\vec{a}(t)\|^2 - \left(\|\vec{v}(t)\|'\right)^2} = \sqrt{e^{2t}+e^{-2t}-(e^{2t}-2+e^{-2t})} = \sqrt{2}$$

5. $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$

$$\vec{v}(t) = -\sin t\,\hat{i} + \cos t\,\hat{j} + \hat{k}$$
$$\|\vec{v}(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$
$$T = \frac{\vec{v}}{\|\vec{v}\|} = -\frac{\sin t}{\sqrt{2}}\hat{i} + \frac{\cos t}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$
$$\vec{a}(t) = -\cos t\,\hat{i} - \sin t\,\hat{j}$$
$$\|\vec{a}(t)\| = \sqrt{\cos^2 t + \sin^2 t} = \sqrt{1} = 1$$

$$\vec{a}_T = \|\vec{v}(t)\|' = (\sqrt{2})' = 0$$
$$\vec{a}_T = \vec{a} \bullet T = \frac{\sin t \cos t}{\sqrt{2}} - \frac{\sin t \cos t}{\sqrt{2}} = 0$$
$$\vec{a}_N = \sqrt{\|\vec{a}(t)\|^2 - \left(\|\vec{v}(t)\|'\right)^2} = \sqrt{1-0} = 1$$