CROSS PRODUCT - ANSWERS

Find the cross product $\vec{u}\times\vec{v}$

1. $\vec{u}=2\hat{i}+3\hat{j}-2\hat{k}$
   $\vec{v}=-4\hat{i}+4\hat{j}+3\hat{k}$

$$\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -2\\ -4 & 4 & 3\end{vmatrix}=\begin{vmatrix}3 & -2\\ 4 & 3\end{vmatrix}\hat{i}-\begin{vmatrix}2 & -2\\ -4 & 3\end{vmatrix}\hat{j}+\begin{vmatrix}2 & 3\\ -4 & 4\end{vmatrix}\hat{k}=17\hat{i}+2\hat{j}+20\hat{k}$$

2. $\vec{u}=4\hat{i}+2\hat{j}+\hat{k}$
   $\vec{v}=-\hat{i}+4\hat{j}-2\hat{k}$

$$\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 2 & 1\\ -1 & 4 & -2\end{vmatrix}=\begin{vmatrix}2 & 1\\ 4 & -2\end{vmatrix}\hat{i}-\begin{vmatrix}4 & 1\\ -1 & -2\end{vmatrix}\hat{j}+\begin{vmatrix}4 & 2\\ -1 & 4\end{vmatrix}\hat{k}=-8\hat{i}+7\hat{j}+18\hat{k}$$

3. $\vec{u}=2\hat{i}+3\hat{j}-2\hat{k}$
   $\vec{v}=2\hat{i}+3\hat{j}-2\hat{k}$

$$\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -2\\ 2 & 3 & -2\end{vmatrix}=\begin{vmatrix}3 & -2\\ 3 & -2\end{vmatrix}\hat{i}-\begin{vmatrix}2 & -2\\ 2 & -2\end{vmatrix}\hat{j}+\begin{vmatrix}2 & 3\\ 2 & 3\end{vmatrix}\hat{k}=0\hat{i}+0\hat{j}+0\hat{k}=\vec{0}$$

4. $\vec{u}=2\hat{i}+3\hat{j}-2\hat{k}$
   $\vec{v}=4\hat{i}+6\hat{j}-4\hat{k}$

$$\vec{u}\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -2\\ 4 & 6 & -4\end{vmatrix}=\begin{vmatrix}3 & -2\\ 6 & -4\end{vmatrix}\hat{i}-\begin{vmatrix}2 & -2\\ 4 & -4\end{vmatrix}\hat{j}+\begin{vmatrix}2 & 3\\ 4 & 6\end{vmatrix}\hat{k}=0\hat{i}+0\hat{j}+0\hat{k}=\vec{0}$$

5. $\vec{u} = 2\hat{i} + 3\hat{j}$
$\vec{v} = -4\hat{i} + 4\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ -4 & 4 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 4 & 0 \end{vmatrix} \hat{i} - \begin{vmatrix} 2 & 0 \\ -4 & 0 \end{vmatrix} \hat{j} + \begin{vmatrix} 2 & 3 \\ -4 & 4 \end{vmatrix} \hat{k} = 0\hat{i} + 0\hat{j} + 20\hat{k} = 20\hat{k}$$

6. $\vec{u} = 5\hat{i} + \hat{k}$
$\vec{v} = 3\hat{i} + 2\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 0 & 1 \\ 3 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 0 & 2 \end{vmatrix} \hat{i} - \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} \hat{j} + \begin{vmatrix} 5 & 0 \\ 3 & 0 \end{vmatrix} \hat{k} = 0\hat{i} - 7\hat{j} + 0\hat{k} = -7\hat{j}$$

7. $\vec{u} = \hat{i}$
$\vec{v} = \hat{j}$

$\vec{u} \times \vec{v} = \hat{k}$

8. $\vec{u} = \hat{j}$
$\vec{v} = \hat{i}$

$\vec{u} \times \vec{v} = -\hat{k}$

9. $\vec{u} = \hat{k}$
$\vec{v} = \hat{i}$

$\vec{u} \times \vec{v} = \hat{j}$

10. $\vec{u} = \hat{i}$
$\vec{v} = \hat{k}$

$\vec{u} \times \vec{v} = -\hat{j}$

11. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then show by direct calculation that $\vec{a} \bullet (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$.

$$\vec{a}\bullet(\vec{b}\times\vec{c}) = (a_1\hat{i}+a_2\hat{j}+a_3\hat{k})\bullet\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix} = (a_1\hat{i}+a_2\hat{j}+a_3\hat{k})\bullet\left(\begin{vmatrix}b_2&b_3\\c_2&c_3\end{vmatrix}\hat{i} - \begin{vmatrix}b_1&b_3\\c_1&c_3\end{vmatrix}\hat{j} + \begin{vmatrix}b_1&b_2\\c_1&c_2\end{vmatrix}\hat{j}\right)$$

$$= \begin{vmatrix}b_2&b_3\\c_2&c_3\end{vmatrix}a_1 - \begin{vmatrix}b_1&b_3\\c_1&c_3\end{vmatrix}a_2 + \begin{vmatrix}b_1&b_2\\c_1&c_2\end{vmatrix}a_3 = \begin{vmatrix}a_1&a_2&a_3\\b_1&b_2&b_3\\c_1&c_2&c_3\end{vmatrix}.$$