DERIVATIVES OF VECTOR-VALUED FUNCTONS - ANSWERS

(1-8) For each vector-valued function $\vec{r}(t)$ below, find $\dfrac{d\vec{r}}{dt}$.

1. $\vec{r}(t) = t\hat{i} + t^2\hat{j} + \frac{1}{t}\hat{k}$

   $\vec{r}'(t) = \hat{i} + 2t\hat{j} - \frac{1}{t^2}\hat{k}$

2. $\vec{r}(t) = (t+1)\hat{i} + (t^2+1)\hat{j} + (t^3+1)\hat{k}$

   $\vec{r}'(t) = \hat{i} + 2t\hat{j} + 3t^2\hat{k}$

3. $\vec{r}(t) = (e^t+1)\hat{i} + (e^{2t}+1)\hat{j} + (te^{t^2}+1)\hat{k}$

   $\vec{r}'(t) = e^t\hat{i} + 2e^{2t}\hat{j} + (2t^2e^{t^2} + e^{t^2})\hat{k}$

4. $\vec{r}(t) = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + t\,\hat{k}$

   $\vec{r}'(t) = -2\sin 2t\,\hat{i} + 2\cos 2t\,\hat{j} + \hat{k}$

5. $\vec{r}(t) = \sqrt{t}\hat{i} + e^{3t}\hat{j} + \ln(t)\hat{k}$

   $\vec{r}'(t) = \frac{1}{2\sqrt{t}}\hat{i} + 3e^{3t}\hat{j} + \frac{1}{t}\hat{k}$

6. $\vec{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j} + \tan(t)\hat{k}$

   $\vec{r}'(t) = -\sin(t)\hat{i} + \cos(t)\hat{j} + \sec^2(t)\hat{k}$

7. $\vec{r}(t) = \frac{t}{1+t^2}\hat{i} + \sec(t)\hat{j} + \frac{e^t - e^{-t}}{2}\hat{k}$

$$\vec{r}'(t) = \frac{1-t^2}{(1+t^2)^2}\hat{i} + \sec(t)\tan(t)\hat{j} + \frac{e^t + e^{-t}}{2}\hat{k}$$

8. $\vec{r}(t) = \cos^2(t)\hat{i} + \sin^2(t)\hat{j} + \sec^2(t)\hat{k}$

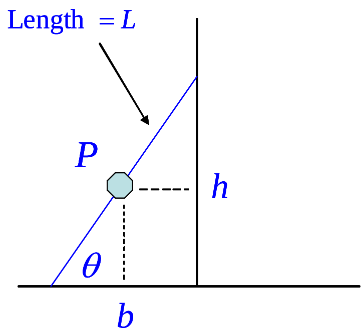
$$\vec{r}'(t) = -2\sin(t)\cos(t)\hat{i} + 2\sin(t)\cos(t)\hat{j} + 2\sec^2(t)\tan(t)\hat{k}$$

9. If $\vec{r}(t)$ is a vector-valued function such that $\|\vec{r}(t)\|$ is constant, then use the derivative with respect to t of $\vec{r}\bullet\vec{r}$ to prove that $\vec{r}(t)$ is perpendicular to $\vec{r}'(t)$.

Suppose that $\|\vec{r}(t)\| = c$ where c is a constant. Then $\|\vec{r}(t)\|^2 = c^2$ is also constant, and $0 = \frac{d(c^2)}{dt} = \frac{d\|\vec{r}(t)\|^2}{dt} = \frac{d(\vec{r}\bullet\vec{r})}{dt} = \vec{r}\bullet\vec{r}' + \vec{r}'\bullet\vec{r} = 2\vec{r}\bullet\vec{r}' \Rightarrow \vec{r}\bullet\vec{r}' = 0 \Rightarrow \vec{r}(t)$ is perpendicular to $\vec{r}'(t)$.

10. A ladder of length L is leaning against a wall, and the top of the ladder is sliding down towards the floor. If the point P is at the midpoint of the ladder and the origin is placed where the wall meets the floor, then find:

   a. The position of P in terms of a vector-valued function $\vec{r}(\theta)$, and
   b. The derivative $\vec{r}'(\theta)$.



Because P is the midpoint of the ladder and because the origin has been placed where the wall meets the floor, it follows that the coordinates of P are $\left(-\frac{b}{2},\frac{h}{2}\right)$. Furthermore, since $\cos\theta=\frac{b}{L}\Rightarrow b=L\cos\theta$ and since $\sin\theta=\frac{h}{L}\Rightarrow h=L\sin\theta$, it follows that the coordinates of P can be written as $\left(-\frac{L\cos\theta}{2},\frac{L\sin\theta}{2}\right)$, and, hence, we can specify the position of P by the vector-valued function $\vec{r}(\theta)=-\frac{L\cos\theta}{2}\hat{i}+\frac{L\sin\theta}{2}\hat{j}$. Therefore,
$\vec{r}'(\theta)=\frac{L\sin\theta}{2}\hat{i}+\frac{L\cos\theta}{2}\hat{j}$.