DIRECTIONAL DERIVATIVES

(1-8) For each of the following functions, find the directional derivative at the point $(1,1)$ in the direction $\vec{u} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$. For problems 1 through 3, also find the maximum and minimum values at $(1,1)$ for any directional derivative. If necessary, round to four decimal places.

1. $z = f(x, y) = x^3 y^2$

2. $z = f(x, y) = \sin(x^3 y^2)$

3. $z = f(x, y) = \sqrt{x^3 y^2}$

4. $z = f(x, y) = \sec(x^3 y^2)$

5. $z = f(x, y) = \tan(x^3 y^2)$

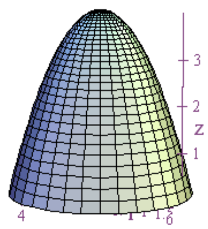
6. $z = f(x, y) = \sin^{-1}(x^3 y^2)$

7. $z = f(x, y) = \dfrac{1}{x^2 + y^2}$

8. $z = f(x, y) = x^2 e^y$

9. For $w = f(x, y, z) = \sin(xyz)$, find the directional derivative at $(1,1,1)$ in the direction of $\vec{u} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$.

10. Suppose you are climbing a hill that is in the shape of the graph of $z = -x^2 - y^2 + 4$, and you are standing at the coordinates $\left(\sqrt{2}, \sqrt{2}\right)$. In terms of unit vectors, what direction should your proceed in if you want to ascend the hill most rapidly? What two directions could you walk in to keep your elevation constant?



11. You are walking on a surface in the desert, and being the mathematician you are, you set up an *xyz*-coordinate system with yourself at the origin and you realize that you can model the terrain by $z = 3x - 2y$. Furthermore, so that you can get some exercise without over exerting yourself, you don't want to walk uphill and you don't want to walk downhill. What direction(s) should you proceed in so that your elevation remains constant? Give your answer(s) in terms of an angle with respect to the positive *x*-axis rounded to the nearest tenth of a degree. Additionally, find the unit vector(s) corresponding to your angle(s), and round the components to the nearest hundredth.