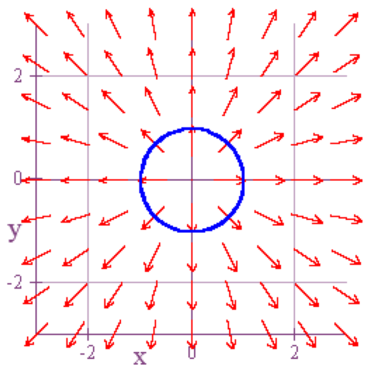
THE DIVERGENCE THEOREM - ANSWERS

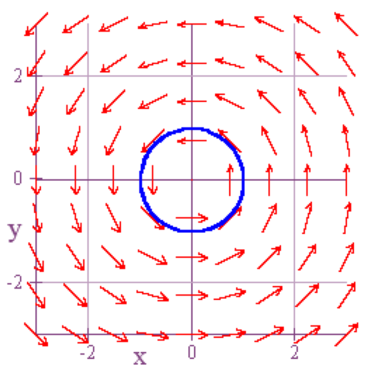
Use the Divergence Theorem (Gauss' Theorem), $Flux = \int_C \vec{F} \bullet N\, ds = \iint_R \nabla \bullet \vec{F}\, dA$, to measure the flux across the boundary of the unit circle (oriented counterclockwise) caused by each of the following vector fields.

1. $\vec{F} = x\hat{i} + y\hat{j}$



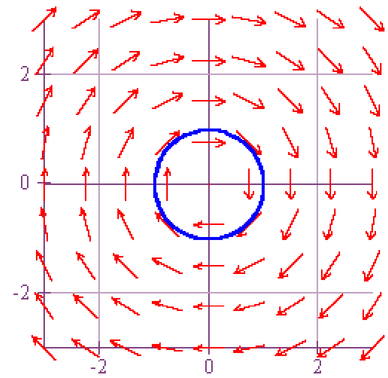
$\iint_R \nabla \bullet \vec{F}\, dA = \iint_R 2\, dA = 2\pi$

2. $\vec{F} = -y\hat{i} + x\hat{j}$



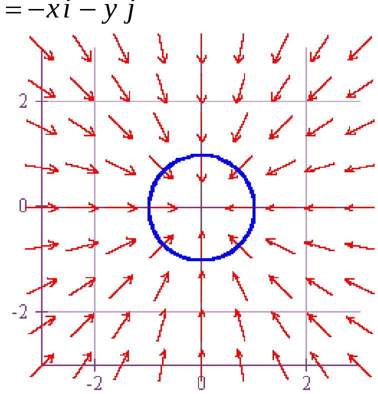
$\iint_R \nabla \bullet \vec{F}\, dA = \iint_R 0\, dA = 0$

3. $\vec{F} = y\hat{i} - x\hat{j}$



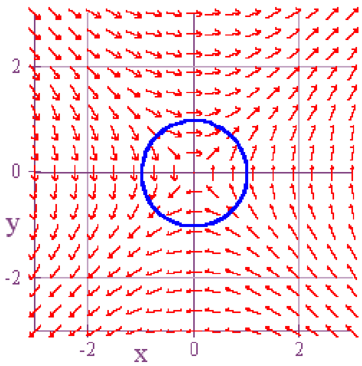
$\iint_R \nabla \bullet \vec{F}\, dA = \iint_R 0\, dA = 0$

4. $\vec{F} = -x\hat{i} - y\hat{j}$



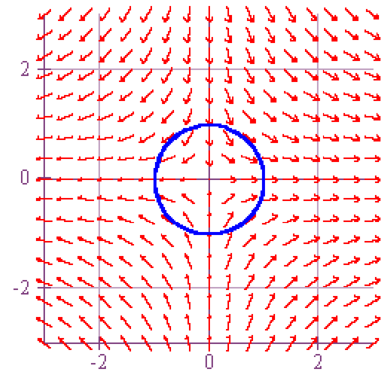
$\iint_R \nabla \bullet \vec{F}\, dA = \iint_R -2\, dA = -2\pi$

5. $\vec{F} = y\hat{i} + x\hat{j}$



$$\iint_R \nabla \bullet \vec{F}\, dA = \iint_R 0\, dA = 0$$

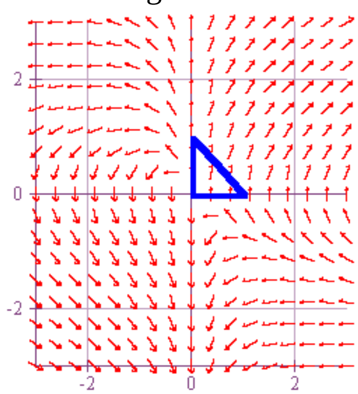
6. $\vec{F} = 4x\hat{i} - 3y\hat{j}$



$$\iint_R \nabla \bullet \vec{F}\, dA = \iint_R (4-3)\, dA = \pi$$

7. $\vec{F} = xy\hat{i} + (x+y)\hat{j}$

R is a triangular region defined by $0 \le x \le 1$ and $0 \le y \le -x+1$, and R is oriented counterclockwise. Use the Divergence Theorem to find the flux across the curve created by the vector field $\vec{F}$.



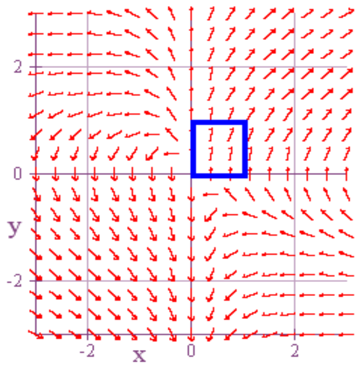
$$\int_C \vec{F} \bullet N\, ds = \iint_R \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dA = \int_0^1 \int_0^{1-x} (y+1)\, dy dx = \int_0^1 \left(\frac{y^2}{2} + y \right)\Bigg|_0^{1-x} dx = \int_0^1 \left(\frac{x^2 - 2x + 1}{2} + 1 - x \right) dx$$

$$= \int_0^1 \left(\frac{x^2 - 4x + 3}{2} \right) dx = \left(\frac{x^3}{6} - x^2 + \frac{3x}{2} \right)\Bigg|_0^1 = \frac{1}{6} - 1 + \frac{3}{2} = \frac{2}{3}$$

8. $\vec{F} = xy\,\hat{i} + (x+y)\,\hat{j}$

R is a square region defined by $0 \le x \le 1$ and $0 \le y \le 1$, and R is oriented counterclockwise.

Use the Divergence Theorem to find the flux across the curve created by the vector field $\vec{F}$.

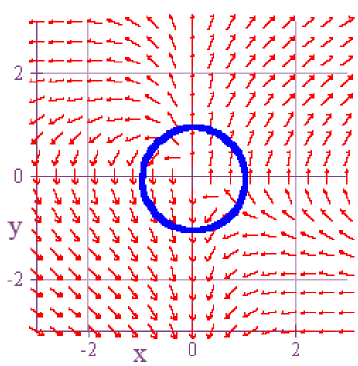


$$\int_C \vec{F} \bullet N\,ds = \iint_R \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dA = \int_0^1\int_0^1 (y+1)\,dy dx = \int_0^1 \left(\frac{y^2}{2} + y\right)\Bigg|_0^1 dx = \int_0^1 \frac{3}{2}dx = \frac{3}{2}x\Big|_0^1 = \frac{3}{2}$$

9. $\vec{F} = xy\,\hat{i} + (x+y)\,\hat{j}$

R is the unit circle, and R is oriented counterclockwise.

Use the Divergence Theorem to find the flux across the curve created by the vector field $\vec{F}$.



$$\int_C \vec{F} \bullet N\,ds = \iint_R \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dA = \iint_R (y+1)\,dA = \int_0^{2\pi}\int_0^1 (r\sin\theta + 1)r\,dr d\theta = \int_0^{2\pi}\int_0^1 (r^2\sin\theta + r)\,dr d\theta$$

$$= \int_0^{2\pi} \left(\frac{r^3}{3}\sin\theta + \frac{r^2}{2}\right)\Bigg|_0^1 d\theta = \int_0^{2\pi}\left(\frac{1}{3}\sin\theta + \frac{1}{2}\right)d\theta = -\frac{\cos\theta}{3} + \frac{\theta}{2}\Bigg|_0^{2\pi} = \pi$$