DOT PRODUCT - ANSWERS

Find the dot product $\vec{u} \bullet \vec{v}$

1. $\vec{u} = 2\hat{i} + 3\hat{j} - 2\hat{k}$
   $\vec{v} = -4\hat{i} + 4\hat{j} + 3\hat{k}$

   $\vec{u} \bullet \vec{v} = -8 + 12 - 6 = -2$

2. $\vec{u} = 4\hat{i} + 2\hat{j} + \hat{k}$
   $\vec{v} = -\hat{i} + 4\hat{j} - 2\hat{k}$

   $\vec{u} \bullet \vec{v} = -4 + 8 - 2 = 2$

3. $\vec{u} = 2\hat{i} + 3\hat{j}$
   $\vec{v} = -4\hat{i} + 4\hat{j}$

   $\vec{u} \bullet \vec{v} = -8 + 12 = 4$

4. $\vec{u} = 5\hat{i} + \hat{j}$
   $\vec{v} = 3\hat{i} + 2\hat{j}$

   $\vec{u} \bullet \vec{v} = 15 + 2 = 17$

5. $\vec{u} = \hat{i}$
   $\vec{v} = \hat{j}$

   $\vec{u} \bullet \vec{v} = 1 \cdot 0 + 0 \cdot 1 = 0$

6. If $\vec{v} = a\,\hat{i} + b\,\hat{j} + c\,\hat{k}$, prove that $\|\vec{v}\|^2 = \vec{v} \bullet \vec{v}$. Give a coherent argument!

   Clearly, $\|\vec{v}\|^2 = \left(\sqrt{a^2 + b^2 + c^2}\right)^2 = a^2 + b^2 + c^2$, and
   $\vec{v} \bullet \vec{v} = (a\,\hat{i} + b\,\hat{j} + c\,\hat{k}) \bullet (a\,\hat{i} + b\,\hat{j} + c\,\hat{k}) = a^2 + b^2 + c^2$. Therefore, $\|\vec{v}\|^2 = \vec{v} \bullet \vec{v}$.

7. Prove or give a counterexample: If you are given three vectors, $\vec{u}$, $\vec{v}$, and $\vec{w}$, and if $\vec{u} \bullet \vec{v} = \vec{u} \bullet \vec{w}$, does it necessarily follow that $\vec{v} = \vec{w}$? Prove or give a counterexample.

   The assertion is not true since, for example, if $\vec{u} = 0\hat{i} + 0\hat{j} + 0\hat{k}$ and if $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$, then $\vec{u} \bullet \vec{v} = \vec{u} \bullet \vec{w}$ but $\vec{v} \neq \vec{w}$.

8. Suppose $\vec{v} = < a, b, c >$ and $\vec{w} = < d, e, f >$, and suppose that for any vector $\vec{u}$ we have $\vec{u} \bullet \vec{v} = \vec{u} \bullet \vec{w}$. Does it necessarily follow that $\vec{v} = \vec{w}$? Prove or give a counterexample.

   If the assertion is true for any vector $\vec{u}$, then it is true for the vectors $\vec{u}_1 = < 1, 0, 0 >$, $\vec{u}_2 = < 0, 1, 0 >$, and $\vec{u}_3 = < 0, 0, 1 >$. Hence,

   $a = \vec{u}_1 \bullet \vec{v} = \vec{u}_1 \bullet \vec{w} = d$

   $b = \vec{u}_2 \bullet \vec{v} = \vec{u}_2 \bullet \vec{w} = e$.

   $c = \vec{u}_3 \bullet \vec{v} = \vec{u}_3 \bullet \vec{w} = f$

   Therefore, $\vec{v} = \vec{w}$ since the corresponding components of the two vectors are the same.