DOUBLE INTEGRALS -ANSWERS

Evaluate the following double integrals.

1. $\iint_R dA$ where R is the rectangle defined by $0 \le x \le 2$ and $0 \le y \le 1$.

$$\iint_R dA = \int_0^2 \int_0^1 dydx = \int_0^2 y\Big|_0^1 dx = \int_0^2 dx = x\Big|_0^2 = 2 - 0 = 2$$

2. $\iint_R dA$ where R is the region enclosed by the curves $y = -x^2 + 1$ and $y = x^2 - 1$.

$$\iint_R dA = \int_{-1}^1 \int_{x^2-1}^{-x^2+1} dydx = \int_{-1}^1 y\Big|_{x^2-1}^{-x^2+1} dx = \int_{-1}^1 \left(-2x^2 + 2\right) dx$$

$$= \frac{-2x^3}{3} + 2x\Big|_{-1}^1 = \left(\frac{-2}{3} + 2\right) - \left(\frac{2}{3} - 2\right) = \frac{8}{3}$$

3. $\iint_R \left(x^2 + y^2\right) dA$ where R is the square defined by $-1 \le x \le 1$ and $-1 \le y \le 1$.

$$\iint_R \left(x^2 + y^2\right) dA = \int_{-1}^1 \int_{-1}^1 \left(x^2 + y^2\right) dydx = \int_{-1}^1 \left(x^2 y + \frac{y^3}{3}\right)\Big|_{-1}^1 dx$$

$$= \int_{-1}^1 \left(x^2 + \frac{1}{3}\right) - \left(-x^2 - \frac{1}{3}\right) dy = \int_{-1}^1 \left(2x^2 + \frac{2}{3}\right) dx = \frac{2x^3}{3} + \frac{2x}{3}\Big|_{-1}^1$$

$$= \left(\frac{2}{3} + \frac{2}{3}\right) - \left(-\frac{2}{3} - \frac{2}{3}\right) = \frac{8}{3}$$

4. $\iint_R (xy)\, dA$ where R is the region defined by $0 \le x \le 1$ and $0 \le y \le x^2$.

$$\iint_R (xy)\, dA = \int_0^1 \int_0^{x^2} xy\, dydx = \int_0^1 \frac{xy^2}{2}\Big|_0^{x^2} dx = \int_0^1 \frac{x^5}{2} dx = \frac{x^6}{12}\Big|_0^1 = \frac{1}{12}$$

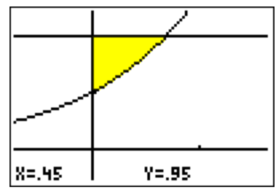
5. $\iint_R x(x+y)dA$ where R is the region defined by $0 \le x \le 1$ and $0 \le y \le 2$.

$$\iint_R x(x+y)dA - \iint_R (x^2+xy)dA = \int_0^1 \int_0^2 (x^2+xy)\,dydx = \int_0^1 (x^2 y + \frac{xy^2}{2})\Big|_0^2 \,dx$$

$$= \int_0^1 (2x^2+2x)\,dx = \left(\frac{2x^3}{3} + x^2\right)\Bigg|_0^1 = \frac{5}{3}$$

6. $\iint_R dA$ where R is the region defined by $0 \le x \le \ln y$ and $1 \le y \le 2$.

We can write our integral in two ways. First,



$$\iint_R dA = \int_1^2 \int_0^{\ln y} dxdy = \int_1^2 x\Big|_0^{\ln y} dy = \int_1^2 \ln y\,dy$$

$$= y\ln y - y\Big|_1^2 = \left(2\ln 2 - 2\right) - \left(1\cdot \ln 1 - 1\right)$$

$$= 2\ln 2 - 1$$

On the other hand, we can reverse the order of integration and have $0 \le x \le \ln 2$ and $e^x \le y \le 2$.

$$\iint_R dA = \int_0^{\ln 2} \int_{e^x}^{2} dydx = \int_0^{\ln 2} y\Big|_{e^x}^{2} dx = \int_0^{\ln 2} \left(2 - e^x\right)dx$$

$$= \left(2x - e^x\right)\Big|_0^{\ln 2} = \left(2\ln 2 - e^{\ln 2}\right) - \left(0 - e^0\right)$$

$$= 2\ln 2 - 1$$

7. $\iint_R \frac{4y}{x^2-1}\,dA$ where R is the region defined by $2 \le x \le 3$ and $0 \le y \le 1$.

$$\iint_R \frac{4y}{x^2-1}\,dA = \int_2^3\int_0^1 \frac{4y}{x^2-1}\,dy\,dx = \int_2^3 2y^2 \cdot \frac{1}{x^2-1}\Bigg|_0^1 dx = \int_2^3 2\left(\frac{A}{x+1}+\frac{B}{x-1}\right)dx$$

$$= \int_2^3 2\left(\frac{-1/2}{x+1}+\frac{1/2}{x-1}\right)dx = \int_2^3 \left(\frac{-1}{x+1}+\frac{1}{x-1}\right)dx = -\ln|x+1| + \ln|x-1|\Bigg|_2^3$$

$$= (-\ln 4 + \ln 2) - (-\ln 3 + \ln 1) = -\ln 4 + \ln 2 + \ln 3 = \ln\left(\frac{2\cdot 3}{4}\right) = \ln\left(\frac{3}{2}\right)$$