

## EQUATIONS OF PLANES - ANSWERS

If possible, for each problem below find the equation for the plane containing the point  $P = (1, 2, 3)$  and the vectors  $\vec{u}$  and  $\vec{v}$ . Write your answer in the form  $z = Ax + By + C$ , if possible.

1.  $\vec{u} = 2\hat{i} + 3\hat{j}$  and  $\vec{v} = 3\hat{i} - 2\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 3 & -2 & 0 \end{vmatrix} = (-4 - 9)\hat{k} = -13\hat{k} = 0\hat{i} + 0\hat{j} - 13\hat{k}$$

$$P = (1, 2, 3)$$

$$0(x-1) + 0(y-2) - 13(z-3) = 0 \Rightarrow -13z + 39 = 0 \Rightarrow z = 3$$

2.  $\vec{u} = 2\hat{i} + 3\hat{j}$  and  $\vec{v} = 4\hat{i} + 3\hat{j}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 4 & 3 & 0 \end{vmatrix} = (6 - 12)\hat{k} = -6\hat{k} = 0\hat{i} + 0\hat{j} - 6\hat{k}$$

$$P = (1, 2, 3)$$

$$0(x-1) + 0(y-2) - 6(z-3) = 0 \Rightarrow -6z + 18 = 0 \Rightarrow z = 3$$

3.  $\vec{u} = \hat{i} + \hat{j} - 5\hat{k}$  and  $\vec{v} = 2\hat{i} + 2\hat{j} - 4\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -5 \\ 2 & 2 & -4 \end{vmatrix} = (-4 + 10)\hat{i} - (-4 + 10)\hat{j} + (2 - 2)\hat{k} = 6\hat{i} - 6\hat{j} + 0\hat{k}$$

$$P = (1, 2, 3)$$

$$6(x-1) - 6(y-2) + 0(z-3) = 0 \Rightarrow 6x - 6y + 6 = 0 \Rightarrow x - y + 1 = 0 \Rightarrow y = x + 1$$

4.  $\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k}$  and  $\vec{v} = 3\hat{i} - 2\hat{j} + \hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 3 & -2 & 1 \end{vmatrix} = (3+2)\hat{i} - (2-3)\hat{j} + (-4-9)\hat{k} = 5\hat{i} + \hat{j} - 13\hat{k}$$

$$P = (1, 2, 3)$$

$$5(x-1) + (y-2) - 13(z-3) = 0 \Rightarrow 5x + y - 13z + 32 = 0 \Rightarrow z = \frac{5}{13}x + \frac{1}{13}y + \frac{32}{13}$$

5.  $\vec{u} = 2\hat{i} + 3\hat{j} + \hat{k}$  and  $\vec{v} = 2\hat{i} + 2\hat{j} - 10\hat{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 2 & 2 & -10 \end{vmatrix} = (-30-2)\hat{i} - (-20-2)\hat{j} + (4-6)\hat{k} = -32\hat{i} + 22\hat{j} - 2\hat{k}$$

$$P = (1, 2, 3)$$

$$-32(x-1) + 22(y-2) - 2(z-3) = 0 \Rightarrow -32x + 22y - 2z - 6 = 0 \Rightarrow z = -16x + 11y - 3$$