FIRST PARTIALS - ANSWERS

For each of the following functions, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

1. $z = f(x, y) = x^3 y^2$

$$\frac{\partial f}{\partial x} = 3x^2 y^2$$

$$\frac{\partial f}{\partial y} = 2x^3 y$$

2. $z = f(x, y) = \sin(x^3 y^2)$

$$\frac{\partial f}{\partial x} = \cos(x^3 y^2) \cdot 3x^2 y^2 = 3x^2 y^2 \cos(x^3 y^2)$$

$$\frac{\partial f}{\partial y} = \cos(x^3 y^2) \cdot 2x^3 y = 2x^3 y \cos(x^3 y^2)$$

3. $z = f(x, y) = \sqrt{x^3 y^2}$

$$\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x^3 y^2}} \cdot 3x^2 y^2 = \frac{3x^2 y^2}{2\sqrt{x^3 y^2}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\sqrt{x^3 y^2}} \cdot 2x^3 y = \frac{x^3 y}{\sqrt{x^3 y^2}}$$

4. $z = f(x, y) = \sec(x^3 y^2)$

$$\frac{\partial f}{\partial x} = \sec(x^3 y^2)\tan(x^3 y^2) \cdot 3x^2 y^2 = 3x^2 y^2 \sec(x^3 y^2)\tan(x^3 y^2)$$

$$\frac{\partial f}{\partial y} = \sec(x^3 y^2)\tan(x^3 y^2) \cdot 2x^3 y = 2x^3 y \sec(x^3 y^2)\tan(x^3 y^2)$$

5. $z = f(x,y) = \tan(x^3 y^2)$

$$\frac{\partial f}{\partial x} = \sec^2(x^3 y^2) \cdot 3x^2 y^2 = 3x^2 y^2 \sec^2(x^3 y^2)$$

$$\frac{\partial f}{\partial y} = \sec^2(x^3 y^2) \cdot 2x^3 y = 2x^3 y \sec^2(x^3 y^2)$$

6. $z = f(x,y) = \sin^{-1}(x^3 y^2)$

$$\frac{\partial f}{\partial x} = \frac{1}{\sqrt{1-(x^3 y^2)^2}} \cdot 3x^2 y^2 = \frac{3x^2 y^2}{\sqrt{1-x^6 y^4}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{\sqrt{1-(x^3 y^2)^2}} \cdot 2x^3 y = \frac{2x^3 y}{\sqrt{1-x^6 y^4}}$$

7. $z = f(x,y) = \sqrt[3]{x^2 + y + 4} = (x^2 + y + 4)^{1/3}$

$$\frac{\partial f}{\partial x} = \frac{1}{3}(x^2 + y + 4)^{-2/3}(2x)$$

$$\frac{\partial f}{\partial y} = \frac{1}{3}(x^2 + y + 4)^{-2/3}$$

8. $z = f(x,y) = e^{-(x^2+y^2)}$

$$\frac{\partial f}{\partial x} = e^{-(x^2+y^2)}(-2x)$$

$$\frac{\partial f}{\partial y} = e^{-(x^2+y^2)}(-2y)$$

9. $z = f(x, y) = \ln(xy)$

$$\frac{\partial f}{\partial x} = \frac{1}{xy} \cdot y = \frac{1}{x}$$

$$\frac{\partial f}{\partial y} = \frac{1}{xy} \cdot x = \frac{1}{y}$$

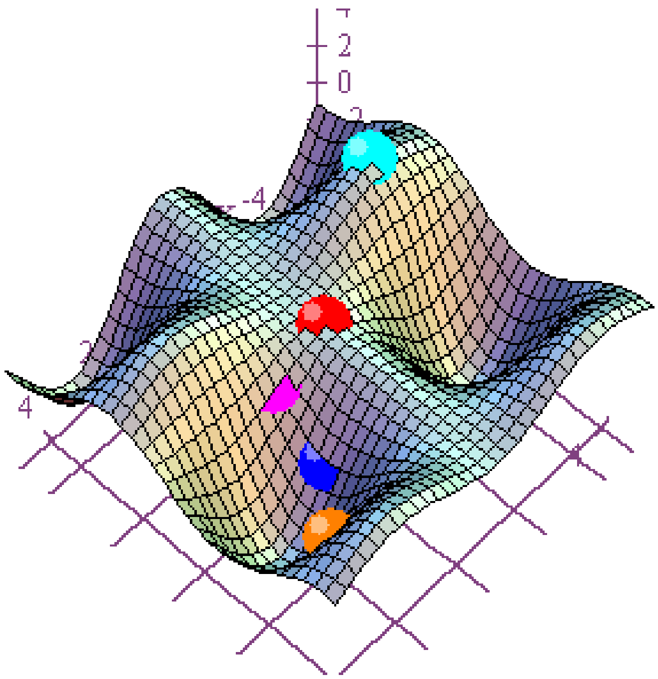
10. $z = f(x, y) = \dfrac{xy+1}{x+y}$

$$\frac{\partial f}{\partial x} = \frac{(x+y)y-(xy+1)}{(x+y)^2} = \frac{xy+y^2-xy-1}{(x+y)^2} = \frac{y^2-1}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{(x+y)x-(xy+1)}{(x+y)^2} = \frac{x^2+xy-xy-1}{(x+y)^2} = \frac{x^2-1}{(x+y)^2}$$

11. Suppose you are walking through a hilly terrain, and you set up an *xyz*-coordinate system with you standing at the point corresponding to $x=0$ and $y=0$ (see the red dot on the graph below). Suppose also that the surface corresponds to the graph of $z=f(x,y)=\cos x-2\sin y+2\sin y\cos x$ with the positive *x*-axis pointing east and the positive *y*-axis pointing north. Then at each of the points given by the *x* and *y* coordinates below, find the rate of change of your elevation in each of the cardinal directions, east, west, north, and south. Also, assume that everything is being measured in feet.

   a. $x=0$ and $y=0$ (the red dot)
   b. $x=\pi$ and $y=\pi$ (the orange dot)
   c. $x=\pi/2$ and $y=\pi/2$ (the blue dot)
   d. $x=\pi/2$ and $y=0$ (the magenta dot)
   e. $x=-\pi$ and $y=-\pi/2$ (the cyan dot)



$z_x=-\sin x-2\sin y\sin x$

$z_y=-2\cos y+2\cos y\cos x$

a. East: 0
   West: 0
   North: 0
   South 0

b. East: 0
   West: 0
   North: 4
   South: -4

c. East: -3
West: 3
North: 0
South: 0

d. East: -1
West: 1
North: -2
South: 2

e. East: 0
West: 0
North: 0
South: 0