GRADIENTS TO LEVEL CURVES AND SURFACES - ANSWERS

For each function below, find parametric equations for the gradient vector at the point below on the indicated level curve or surface.

1. $z = x^2 + y^2$, level curve $z = 1$, $P = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$.

$\nabla z = 2x\hat{i} + 2y\hat{j}$

$\nabla z\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = \sqrt{2}\hat{i} + \sqrt{2}\,\hat{j}$

$x = \frac{1}{\sqrt{2}} + t\sqrt{2}$

$y = \frac{1}{\sqrt{2}} + t\sqrt{2}$

$0 \le t \le 1$

2. $z = x^2 - y^2$, level curve $z = 1$, $P = (1,0)$.

$\nabla z = 2x\hat{i} - 2y\hat{j}$

$\nabla z(1,0) = 2\hat{i}$

$x = 1 + 2t$

$y = 0$

$0 \le t \le 1$

3. $z = -x^2 - y^2$, level curve $z = -1$, $P = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$.

$\nabla z = -2x\hat{i} - 2y\hat{j}$

$\nabla z\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = -\sqrt{2}\hat{i} - \sqrt{2}\,\hat{j}$

$x = \frac{1}{\sqrt{2}} - t\sqrt{2}$

$y = \frac{1}{\sqrt{2}} - t\sqrt{2}$

$0 \le t \le 1$

4. $z = x^2 + y^2 - 1$, level surface $x^2 + y^2 - z = 1$, $P = (1,1,1)$.

$w = x^2 + y^2 - z$
$\nabla w = 2x\hat{i} + 2y\hat{j} - \hat{k}$
$\nabla w(1,1,1) = 2\hat{i} + 2\hat{j} - \hat{k}$

$x = 1 + 2t$
$y = 1 + 2t$
$z = 1 - t$
$0 \le t \le 1$

5. $z = x^2 - y^2 + 1$, level surface $x^2 - y^2 - z = -1$, $P = (1,1,1)$.

$w = x^2 - y^2 - z$
$\nabla w = 2x\hat{i} - 2y\hat{j} - \hat{k}$
$\nabla w(1,1,1) = 2\hat{i} - 2\hat{j} - \hat{k}$

$x = 1 + 2t$
$y = 1 - 2t$
$z = 1 - t$
$0 \le t \le 1$

6. $w = x^2 + y^2 + z^2$, level surface $w = 3$, $P = (1,1,1)$.

$w = x^2 + y^2 + z^2$
$\nabla w = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$
$\nabla w(1,1,1) = 2\hat{i} + 2\hat{j} + 2\hat{k}$

$x = 1 + 2t$
$y = 1 + 2t$
$z = 1 + 2t$
$0 \le t \le 1$