GRADIENT TO A LEVEL CURVE - ANSWERS

For each of the following functions, find ∇z, the gradient.

1. $z = f(x,y) = x^3y^2$

$$\nabla z = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = 3x^2y^2\,\hat{i} + 2x^3y\,\hat{j} = \left\langle 3x^2y^2, 2x^3y \right\rangle$$

2. $z = f(x,y) = \sin(x^3y^2)$

$$\begin{aligned}\nabla z &= \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \cos(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \cos(x^3y^2)\cdot 2x^3y\,\hat{j} \\ &= \left\langle 3\cos(x^3y^2)x^2y^2, 2\cos(x^3y^2)x^3y \right\rangle\end{aligned}$$

3. $z = f(x,y) = \sqrt{x^3y^2}$

$$\begin{aligned}\nabla z &= \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \frac{1}{2\sqrt{x^3y^2}}\cdot 3x^2y^2\,\hat{i} + \frac{1}{2\sqrt{x^3y^2}}\cdot 2x^3y\,\hat{j} \\ &= \left\langle 3\frac{1}{2\sqrt{x^3y^2}}x^2y^2, 2\frac{1}{2\sqrt{x^3y^2}}x^3y \right\rangle = \left\langle \frac{3x^2y^2}{2\sqrt{x^3y^2}}, \frac{x^3y}{\sqrt{x^3y^2}} \right\rangle\end{aligned}$$

4. $z = f(x,y) = \sec(x^3y^2)$

$$\begin{aligned}\nabla z &= \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \sec(x^3y^2)\tan(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \sec(x^3y^2)\tan(x^3y^2)\cdot 2x^3y\,\hat{j} \\ &= \left\langle 3\sec(x^3y^2)\tan(x^3y^2)x^2y^2, 2\sec(x^3y^2)\tan(x^3y^2)x^3y \right\rangle\end{aligned}$$

5. $z = f(x,y) = \tan(x^3y^2)$

$$\begin{aligned}\nabla z &= \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \sec^2(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \sec^2(x^3y^2)\cdot 2x^3y\,\hat{j} \\ &= \left\langle 3\sec^2(x^3y^2)x^2y^2, 2\sec^2(x^3y^2)x^3y \right\rangle\end{aligned}$$

6. $z = f(x,y) = \sin^{-1}(x^3y^2)$

$$\nabla z = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \frac{1}{\sqrt{1-(x^3y^2)^2}} \cdot 3x^2y^2\,\hat{i} + \frac{1}{\sqrt{1-(x^3y^2)^2}} \cdot 2x^3y\,\hat{j}$$

$$= \left\langle 3\frac{1}{\sqrt{1-x^6y^4}}x^2y^2, 2\frac{1}{\sqrt{1-x^6y^4}}x^3y \right\rangle = \left\langle \frac{3x^2y^2}{\sqrt{1-x^6y^4}}, \frac{2x^3y}{\sqrt{1-x^6y^4}} \right\rangle$$

7. $z = f(x,y) = \sqrt[3]{x^2+y+4} = (x^2+y+4)^{1/3}$

$$\nabla z = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \frac{2x}{3(x^2+y+4)^{2/3}}\hat{i} + \frac{1}{3(x^2+y+4)^{2/3}}\hat{j}$$

8. $z = f(x,y) = e^{-(x^2+y^2)}$

$$\nabla z = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \frac{-2x}{e^{(x^2+y^2)}}\hat{i} + \frac{-2y}{e^{(x^2+y^2)}}\hat{j}$$

9. $z = f(x,y) = \ln(xy)$

$$\nabla z = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \frac{1}{x}\hat{i} + \frac{1}{y}\hat{j}$$

10. $z = f(x,y) = \dfrac{xy+1}{x+y}$

$$\nabla z = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = \frac{y^2-1}{(x+y)^2}\hat{i} + \frac{x^2-1}{(x+y)^2}\hat{j}$$