GRADIENT TO A LEVEL SURFACE - ANSWERS

(1-6) For each of the following functions, let $w = f(x,y) - z$ and find ∇w, the gradient.

1. $z = f(x,y) = x^3y^2$

$$w = x^3y^2 - z$$
$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = 3x^2y^2\,\hat{i} + 2x^3y\,\hat{j} - \hat{k} = \left\langle 3x^2y^2, 2x^3y, -1 \right\rangle$$

2. $z = f(x,y) = \sin(x^3y^2)$

$$w = f(x,y) - z = \sin(x^3y^2) - z$$
$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = \cos(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \cos(x^3y^2)\cdot 2x^3y\,\hat{j} - \hat{k}$$
$$= \left\langle 3\cos(x^3y^2)x^2y^2, 2\cos(x^3y^2)x^3y, -1 \right\rangle$$

3. $z = f(x,y) = \sqrt{x^3y^2}$

$$w = f(x,y) - z = \sqrt{x^3y^2} - z$$
$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = \frac{1}{2\sqrt{x^3y^2}}\cdot 3x^2y^2\,\hat{i} + \frac{1}{2\sqrt{x^3y^2}}\cdot 2x^3y\,\hat{j} - \hat{k}$$
$$= \left\langle 3\frac{1}{2\sqrt{x^3y^2}}x^2y^2, 2\frac{1}{2\sqrt{x^3y^2}}x^3y, -1 \right\rangle = \left\langle \frac{3x^2y^2}{2\sqrt{x^3y^2}}, \frac{x^3y}{\sqrt{x^3y^2}}, -1 \right\rangle$$

4. $z = f(x,y) = \sec(x^3y^2)$

$$w = f(x,y) - z = \sec(x^3y^2) - z$$
$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = \sec(x^3y^2)\tan(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \sec(x^3y^2)\tan(x^3y^2)\cdot 2x^3y\,\hat{j} - \hat{k}$$
$$= \left\langle 3\sec(x^3y^2)\tan(x^3y^2)x^2y^2, 2\sec(x^3y^2)\tan(x^3y^2)x^3y, -1 \right\rangle$$

5. $z = f(x,y) = \tan(x^3y^2)$

$$w = f(x,y) - z = \tan(x^3y^2) - z$$
$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = \sec^2(x^3y^2)\cdot 3x^2y^2\,\hat{i} + \sec^2(x^3y^2)\cdot 2x^3y\,\hat{j} - \hat{k}$$
$$= \left\langle 3\sec^2(x^3y^2)x^2y^2, 2\sec^2(x^3y^2)x^3y, -1 \right\rangle$$

6. $z = f(x,y) = \sin^{-1}(x^3y^2)$

$$w = f(x,y) - z = \sin^{-1}(x^3y^2) - z$$
$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = \frac{1}{\sqrt{1-(x^3y^2)^2}}\cdot 3x^2y^2\,\hat{i} + \frac{1}{\sqrt{1-(x^3y^2)^2}}\cdot 2x^3y\,\hat{j} - \hat{k}$$
$$= \left\langle 3\frac{1}{\sqrt{1-x^6y^4}}x^2y^2, 2\frac{1}{\sqrt{1-x^6y^4}}x^3y, -1 \right\rangle = \left\langle \frac{3x^2y^2}{\sqrt{1-x^6y^4}}, \frac{2x^3y}{\sqrt{1-x^6y^4}}, -1 \right\rangle$$

7. If $w = f(x,y,z) = \sin(xyz)$, find ∇w.

$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = \cos(xyz)\cdot yz\,\hat{i} + \cos(xyz)\cdot xz\,\hat{j} + \cos(xyz)\cdot xy\,\hat{k}$$

8. If $w = f(x,y,z) = x^2 + y^2 + z^2$, find ∇w.

$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = 2x\,\hat{i} + 2y\,\hat{j} + 2z\,\hat{k}$$

9. If $w = f(x,y,z) = x^2e^{yz}$, find ∇w.

$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = 2xe^{yz}\,\hat{i} + x^2ze^{yz}\,\hat{j} + x^2ye^{yz}\,\hat{k}$$

10. If $w = f(x,y,z) = \sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{1/2}$, find ∇w

$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k}$$

$$= \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2x)\hat{i} + \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2y)\hat{j} + \frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2z)\hat{k}$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}$$