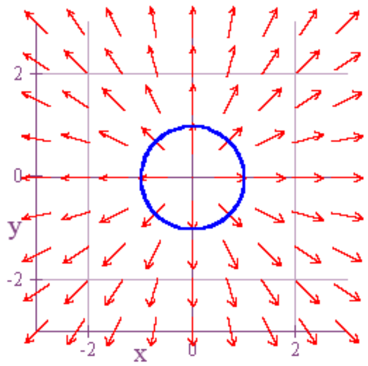
GREEN'S THEOREM AND STOKES' THEOREM

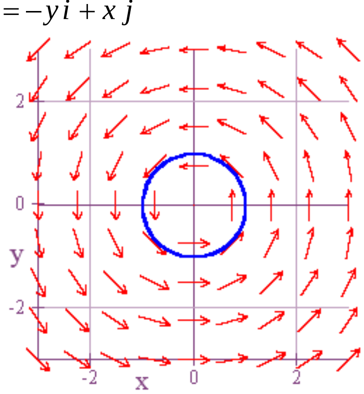
Use Green's Theorem (which in 2-dimensions is the same as Stokes' Theorem),

$Circulation = \int_C \vec{F} \bullet T\, ds = \int_C \vec{F} \bullet d\vec{r} = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \iint_R \left(\nabla \times \vec{F} \right) \bullet \hat{k}\, dA$, to measure the circulation around the boundary of the unit circle (oriented counterclockwise) caused by each of the following vector fields.

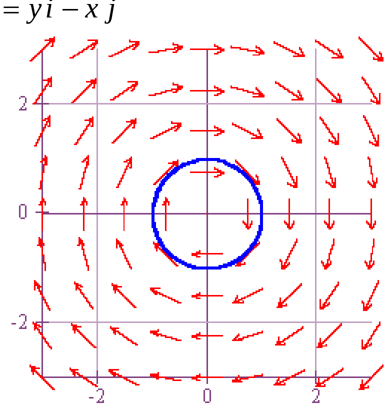
1. $\vec{F} = x\hat{i} + y\hat{j}$



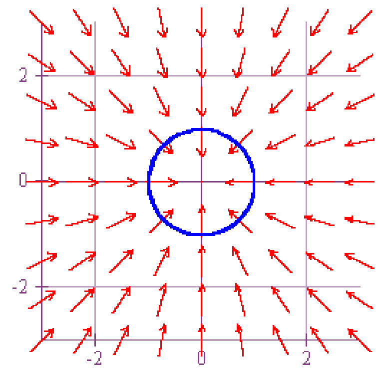
2. $\vec{F} = -y\hat{i} + x\hat{j}$



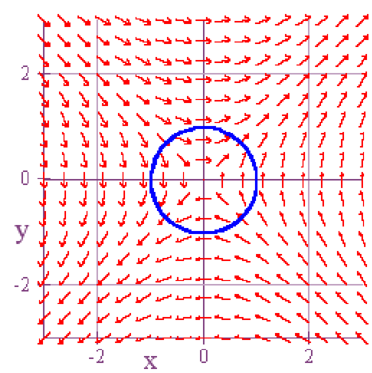
3. $\vec{F} = y\hat{i} - x\hat{j}$



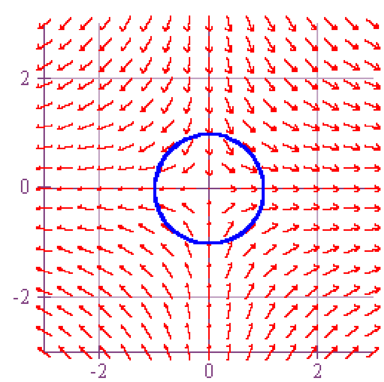
4. $\vec{F} = -x\hat{i} - y\hat{j}$



5. $\vec{F} = y\hat{i} + x\hat{j}$



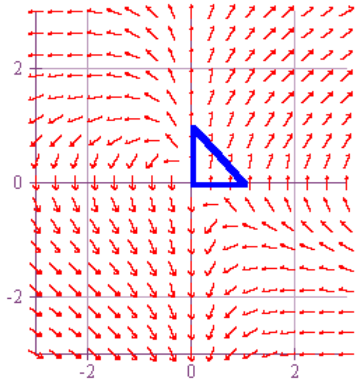
6. $\vec{F} = 4x\hat{i} - 3y\hat{j}$



7. $\vec{F} = xy\,\hat{i} + (x+y)\,\hat{j}$

R is a triangular region defined by $0 \le x \le 1$ and $0 \le y \le -x+1$, and R is oriented counterclockwise.

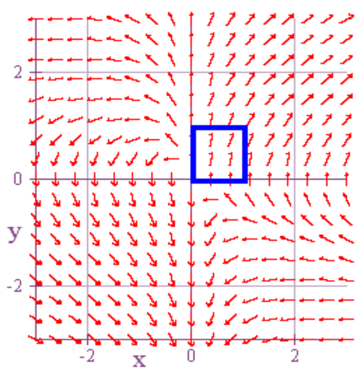
Use Green's Theorem to find the circulation around the curve created by the vector field $\vec{F}$.



8. $\vec{F} = xy\,\hat{i} + (x+y)\,\hat{j}$

R is a square region defined by $0 \le x \le 1$ and $0 \le y \le 1$, and R is oriented counterclockwise.

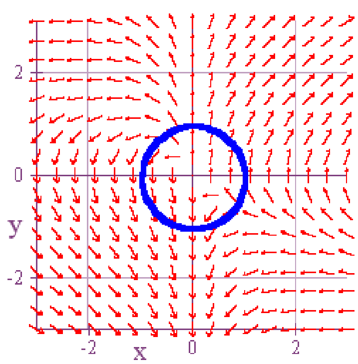
Use Green's Theorem to find the circulation around the curve created by the vector field $\vec{F}$.



9. $\vec{F} = xy\,\hat{i} + (x+y)\,\hat{j}$

R is the unit circle, and R is oriented counterclockwise.

Use Green's Theorem to find the circulation around the curve created by the vector field $\vec{F}$.



10. Evaluate $\int_C ye^{-x}dx + (x^2/2 - e^{-x})dy$ where C is the circle of radius 1 with center at (2,0), oriented counterclockwise. (HINT: Use a change a variables to move the center to the origin, and then integrate using polar coordinates.)