INTEGRALS OF VECTOR-VALUED FUNCTONS - ANSWERS

For each vector-valued function $\vec{r}(t)$ below, find $\int \vec{r}(t)\,dt$.

1. $\vec{r}(t) = t\hat{i} + t^2\hat{j} + \frac{1}{t}\hat{k}$

$$\int \vec{r}(t)\,dt = \frac{t^2}{2}\hat{i} + \frac{t^3}{3}\hat{j} + \ln|t|\hat{k} + \vec{C}$$

2. $\vec{r}(t) = (t+1)\hat{i} + (t^2+1)\hat{j} + (t^3+1)\hat{k}$

$$\int \vec{r}(t)\,dt = \left(\frac{t^2}{2} + t\right)\hat{i} + \left(\frac{t^3}{3} + t\right)\hat{j} + \left(\frac{t^4}{4} + t\right)\hat{k} + \vec{C}$$

3. $\vec{r}(t) = (e^t+1)\hat{i} + (e^{2t}+1)\hat{j} + (te^{t^2}+1)\hat{k}$

$$\int \vec{r}(t)\,dt = \left(e^t + t\right)\hat{i} + \left(\frac{e^{2t}}{2} + t\right)\hat{j} + \left(\frac{e^{t^2}}{2} + t\right)\hat{k} + \vec{C}$$

4. $\vec{r}(t) = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + t\,\hat{k}$

$$\int \vec{r}(t)\,dt = \frac{\sin 2t}{2}\hat{i} - \frac{\cos 2t}{2}\hat{j} + \frac{t^2}{2}\hat{k} + \vec{C}$$

5. $\vec{r}(t) = \sqrt{t}\hat{i} + e^{3t}\hat{j} + \ln(t)\hat{k}$

$$\int \vec{r}(t)\,dt = \frac{2t^{3/2}}{3}\hat{i} + \frac{e^{3t}}{3}\hat{j} + (t\ln t - t)\hat{k} + \vec{C}$$

6. $\vec{r}(t) = \cos(t)\hat{i} + \sin(t)\hat{j} + \tan(t)\hat{k}$

$$\int \vec{r}(t)\,dt = \sin(t)\hat{i} - \cos(t)\hat{j} - \ln|\cos t|\hat{k} + \vec{C}$$

7. $\vec{r}(t) = \dfrac{t}{1+t^2}\hat{i} + \sec(t)\,\hat{j} + \dfrac{e^t - e^{-t}}{2}\hat{k}$

$$\int \vec{r}(t)\,dt = \frac{1}{2}\ln(1+t^2)\hat{i} + \ln\left|\sec t + \tan t\right|\hat{j} + \frac{e^t + e^{-t}}{2}\hat{k} + \vec{C}$$

8. $\vec{r}(t) = \cos^2(t)\hat{i} + \sin^2(t)\,\hat{j} + \sec^2(t)\hat{k}$

$$\int \vec{r}(t)\,dt = \left(\frac{t}{2} + \frac{\sin 2t}{4}\right)\hat{i} + \left(\frac{t}{2} - \frac{\sin 2t}{4}\right)\hat{j} + \tan(t)\hat{k} + \vec{C}$$