JOINT DENSITY FUNCTIONS - ANSWERS

(1-3) Let $p(x,y)=\begin{cases}\frac{3}{2}x+3y & if\;\; 0\le x\le 1 \;and\; 0\le y\le x\\ 0 & elsewhere\end{cases}$ be a joint density function.

1. Find the probability that $\frac{1}{2}\le x\le 1$ and $0\le y\le \frac{1}{2}$.

$$\int_{1/2}^{1}\int_{0}^{1/2} p(x,y)\,dydx=\int_{1/2}^{1}\int_{0}^{1/2}\left(\frac{3}{2}x+3y\right)dydx=\int_{1/2}^{1}\left(\frac{3}{2}xy+\frac{3y^2}{2}\right)\Bigg|_{0}^{1/2} dx$$

$$=\int_{1/2}^{1}\left(\frac{3x}{4}+\frac{3}{8}\right)dx=\left(\frac{3x^2}{8}+\frac{3x}{8}\right)\Bigg|_{1/2}^{1}=\left(\frac{3}{8}+\frac{3}{8}\right)-\left(\frac{3}{32}+\frac{3}{16}\right)$$

$$=\frac{15}{32}=0.46875$$

2. Find the probability that $\frac{1}{2}\le x\le 1$ and $0\le y\le x$.

$$\int_{1/2}^{1}\int_{0}^{x} p(x,y)\,dydx=\int_{1/2}^{1}\int_{0}^{x}\left(\frac{3}{2}x+3y\right)dydx=\int_{1/2}^{1}\left(\frac{3}{2}xy+\frac{3y^2}{2}\right)\Bigg|_{0}^{x} dx$$

$$=\int_{1/2}^{1}\left(\frac{3x^2}{2}+\frac{3x^2}{2}\right)dx=\int_{1/2}^{1}3x^2\,dx=x^3\Bigg|_{1/2}=1-\frac{1}{8}=\frac{7}{8}=0.875$$

3. Find the probability that $0\le y\le \frac{1}{2}$ and $y\le x\le \frac{1}{2}$.

$$\int_{0}^{1/2}\int_{y}^{1/2} p(x,y)\,dxdy=\int_{0}^{1/2}\int_{y}^{1/2}\left(\frac{3}{2}x+3y\right)dxdy=\int_{0}^{1/2}\left(\frac{3x^2}{4}+3xy\right)\Bigg|_{y}^{1/2} dy$$

$$=\int_{0}^{1/2}\left(\frac{3}{16}+\frac{3y}{2}\right)-\left(\frac{3y^2}{4}+3y^2\right)dy=\frac{3y}{16}+\frac{3y^2}{4}-\frac{y^3}{4}-y^3\Bigg|_{0}^{1/2}$$

$$=\frac{3}{32}+\frac{3}{16}-\frac{1}{32}-\frac{1}{8}=\frac{1}{8}=0.125$$

4. If $p(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$ is a normal distribution with $\mu=0$ and $\sigma=1$ and if $q(y)=\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}}$ is another normal distribution with $\mu=0$ and $\sigma=1$, then find the probability that $-1\le x\le 1$ and $-1\le y\le 1$. Set up a double integral and use **fnInt** on your TI-83/84 calculator to approximate numerically rounding to the nearest hundredth.

$$\int_{-1}^{1}\int_{-1}^{1}\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}\cdot\frac{1}{\sqrt{2\pi}}e^{\frac{-y^2}{2}}\,dydx=\int_{-1}^{1}\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}\,dx\cdot\int_{-1}^{1}\frac{1}{\sqrt{2\pi}}e^{\frac{-y^2}{2}}\,dy$$
$$\approx 0.6827\cdot 0.6827\approx 0.4661\approx 0.47$$

5. If the weights of adult men are normally distributed with a mean of 200 pounds and a standard deviation of 10 pounds, and if IQ is normally distributed with a mean of 100 and a standard deviation of 15 points, then what is the probability that an adult male has a weight between 200 and 210 pounds and an IQ between 100 and 120? Let *x* equal weight and *y* equal IQ, set up a double integral, and use **fnInt** on your TI-83/84 calculator to approximate numerically rounding to the nearest hundredth. Use the general formula for a normal distribution, $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$.

$$\int_{200}^{210}\int_{100}^{120}\frac{1}{10\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-200}{10}\right)^2}\cdot\frac{1}{15\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y-100}{15}\right)^2}\,dydx$$
$$=\int_{200}^{210}\frac{1}{10\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-200}{10}\right)^2}\,dx\cdot\int_{100}^{120}\frac{1}{15\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{y-100}{15}\right)^2}\,dy\approx 0.3413\cdot 0.4088\approx 0.1395\approx 0.14$$