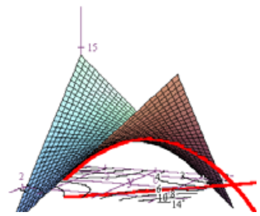
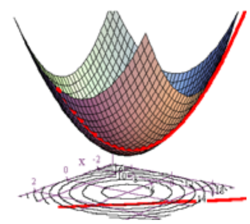
LAGRANGE MULTIPLIERS

Use the method of Lagrange multipliers to solve the following problems.

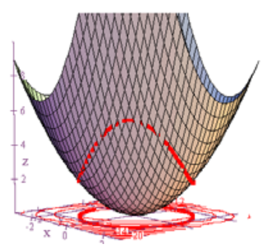
1. Find the coordinates of the maximum point on the graph of $z = xy + 5$ subject to the constraint $x + y = 2$.

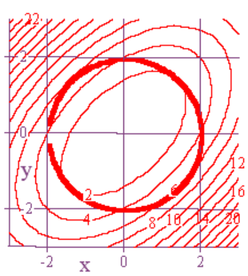


2. Find the coordinates of the minimum point on the graph of $z = x^2 + y^2 + 5$ subject to the constraint $x + y = 2$.



3. Find the coordinates of the extreme points on the graph of $z = x^2 - xy + y^2$ subject to the constraint $x^2 + y^2 = 4$.





4. Let $w = xyz$ for $x > 0$, $y > 0$, and $z > 0$. Find the maximum value of w subject to the constraint $x + y + z = 48$.

5. A manufacturer has an order for 1000 ultra-deluxe time machines with built-in MP3 player. Suppose the units are manufactured in two different locations with x representing the number of units produced in one location and y the number of units in the other. If the total cost of production is given by $z = C(x, y) = x^2 + 10x + 0.50y^2 + 12y - 10{,}000$ *dollars*, find the values of x and y that will minimize the costs and find the minimum cost.

6. Find the points on the circle $x^2 + y^2 = 100$ that are closest to and farthest from the point $(2,3)$.

7. Find the area of the largest rectangle that can be inscribed inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

8. Find the volume of the largest rectangular box that can be inscribed inside the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

9. Use Lagrange multipliers to find the point in the plane $2x + y - z = -5$ that is closest to the origin. (HINT: Minimize the square of the distance from the origin. You will get the same answer, but you won't have to mess with derivatives of square roots.)

10. A company operates two plants which manufacture the same item. Suppose that the total cost involved in producing quantities q_1 and q_2 at the two plants is $C = 2q_1{}^2 + q_1q_2 + q_2{}^2 + 1000$. Suppose also that the company's objective is to produce a total quantity of $q_1 + q_2 = 100$ units. Find levels of production, q_1 and q_2, that will minimize the cost.