LENGTH OF A VECTOR - ANSWERS

1. Find the length of $\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

   $\|\vec{u}\| = \sqrt{29} \approx 5.3852$

2. Find the length of $\vec{v} = \hat{i} - 5\hat{j} + \hat{k}$.

   $\|\vec{v}\| = \sqrt{27} \approx 5.1962$

3. Find the length of $\vec{w} = -3\hat{i} - 2\hat{j} - 8\hat{k}$.

   $\|\vec{w}\| = \sqrt{77} \approx 8.775$

4. Find the length of $\vec{v} = 3\hat{i} + 2\hat{j} - 2\hat{k}$.

   $\|\vec{v}\| = \sqrt{17} \approx 4.1231$

5. Find a unit vector pointing in the same direction as $\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

   $$\frac{\vec{u}}{\|\vec{u}\|} = \frac{2}{\sqrt{29}}\hat{i} + \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k} = \frac{2\sqrt{29}}{29}\hat{i} + \frac{3\sqrt{29}}{29}\hat{j} + \frac{4\sqrt{29}}{29}\hat{k}$$

6. Find a unit vector pointing in the opposite direction as $\vec{v} = \hat{i} - 5\hat{j} + \hat{k}$.

   $$-\frac{\vec{v}}{\|\vec{v}\|} = -\frac{1}{\sqrt{27}}\hat{i} + \frac{5}{\sqrt{27}}\hat{j} - \frac{1}{\sqrt{27}}\hat{k} = -\frac{\sqrt{27}}{27}\hat{i} + \frac{5\sqrt{27}}{27}\hat{j} - \frac{\sqrt{27}}{27}\hat{k}$$

7. Find a vector of length 2 in the direction of $\vec{v} = \hat{i} - 5\hat{j} + \hat{k}$.

   $$2 \cdot \frac{\vec{v}}{\|\vec{v}\|} = \frac{2}{\sqrt{27}}\hat{i} - \frac{10}{\sqrt{27}}\hat{j} + \frac{2}{\sqrt{27}}\hat{k} = \frac{2\sqrt{27}}{27}\hat{i} - \frac{10\sqrt{27}}{27}\hat{j} + \frac{2\sqrt{27}}{27}\hat{k}$$

8. Find a vector of length 10 in the direction of $\vec{w} = -3\hat{i} - 2\hat{j} - 8\hat{k}$.

$$10 \cdot \frac{\vec{w}}{\|\vec{w}\|} = -\frac{30}{\sqrt{77}}\hat{i} - \frac{20}{\sqrt{77}}\hat{j} - \frac{80}{\sqrt{77}}\hat{k} = -\frac{30\sqrt{77}}{77}\hat{i} - \frac{20\sqrt{77}}{77}\hat{j} - \frac{80\sqrt{77}}{77}\hat{k}$$

9. Find a vector of length 2 in the direction opposite of $\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

$$-2 \cdot \frac{\vec{u}}{\|\vec{u}\|} = -\frac{4}{\sqrt{29}}\hat{i} - \frac{6}{\sqrt{29}}\hat{j} - \frac{8}{\sqrt{29}}\hat{k} = -\frac{4\sqrt{29}}{29}\hat{i} - \frac{6\sqrt{29}}{29}\hat{j} - \frac{8\sqrt{29}}{29}\hat{k}$$

10. Find a unit vector pointing in the same direction as $\vec{w} = -3\hat{i} - 2\hat{j} - 8\hat{k}$.

$$\frac{\vec{w}}{\|\vec{w}\|} = -\frac{3}{\sqrt{77}}\hat{i} - \frac{2}{\sqrt{77}}\hat{j} - \frac{8}{\sqrt{77}}\hat{k} = -\frac{3\sqrt{77}}{77}\hat{i} - \frac{2\sqrt{77}}{77}\hat{j} - \frac{8\sqrt{77}}{77}\hat{k}$$

11. If $\vec{u} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, find the unit vector in the direction of $-\vec{u}$.

$$-\frac{\vec{u}}{\|\vec{u}\|} = -\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} - \frac{4}{\sqrt{29}}\hat{k} = -\frac{2\sqrt{29}}{29}\hat{i} - \frac{3\sqrt{29}}{29}\hat{j} - \frac{4\sqrt{29}}{29}\hat{k}$$

12. Prove that if $\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\vec{w} = 3a\hat{i} + 3b\hat{j} + 3c\hat{k}$, then $\|\vec{w}\| = 3\|\vec{v}\|$.

Clearly, $3\|\vec{v}\| = 3\sqrt{a^2 + b^2 + c^2}$ and
$\|\vec{w}\| = \sqrt{9a^2 + 9b^2 + 9c^2} = \sqrt{9(a^2 + b^2 + c^2)} = 3\sqrt{a^2 + b^2 + c^2}$. Therefore, $\|\vec{w}\| = 3\|\vec{v}\|$.