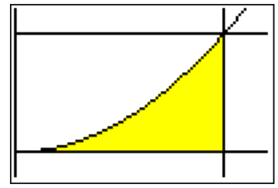
ORDER OF INTEGRATION - ANSWERS

Evaluate the following integrals by reversing the order of integration.

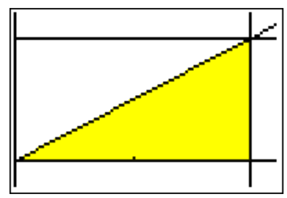
1. $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\ dxdy$



$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\ dxdy = \int_0^1 \int_0^{x^2} \sqrt{x^3+1}\ dydx = \int_0^1 y\sqrt{x^3+1}\Big|_0^{x^2}\ dx$$

$$= \int_0^1 x^2\sqrt{x^3+1}\,dx = \frac{2\left(x^3+1\right)^{3/2}}{9}\Bigg|_0^1 = \frac{2\cdot 2^{3/2}}{9} - \frac{2}{9} = \frac{4\sqrt{2}-2}{9}$$

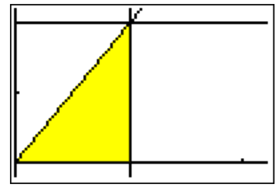
2. $\int_0^1 \int_{2y}^2 \sqrt{x^2+1}\ dxdy$



$$\int_0^1 \int_{2y}^2 \sqrt{x^2+1}\ dxdy = \int_0^2 \int_0^{x/2} \sqrt{x^2+1}\ dydx = \int_0^2 y\sqrt{x^2+1}\Big|_0^{x/2}\ dx$$

$$= \int_0^2 \frac{x}{2}\sqrt{x^2+1}\,dx = \frac{\left(x^2+1\right)^{3/2}}{6}\Bigg|_0^2 = \frac{5^{3/2}}{6} - \frac{1}{6} = \frac{5\sqrt{5}-1}{6}$$

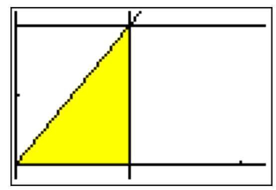
3. $\int_0^2 \int_{\frac{y}{2}}^1 \sin(x^2)\ dxdy$



$$\int_0^2 \int_{\frac{y}{2}}^1 \sin(x^2)\ dxdy = \int_0^1 \int_0^{2x} \sin(x^2)\ dydx = \int_0^1 y\sin(x^2)\Big|_0^{2x}\ dx$$

$$= \int_0^1 2x\sin(x^2)\,dx = -\cos(x^2)\Big|_0^1 = -\cos(1)+\cos(0) = 1-\cos(1) \approx 0.4597$$

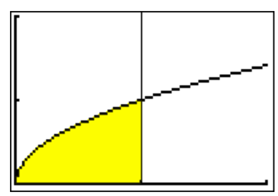
4. $\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\ dxdy$



$$\int_0^2 \int_{\frac{y}{2}}^1 e^{x^2}\ dxdy = \int_0^1 \int_0^{2x} e^{x^2}\ dydx = \int_0^1 ye^{x^2}\Big|_0^{2x}\ dx = \int_0^1 2xe^{x^2}\ dx$$

$$= e^{x^2}\Big|_0^1 = e-1$$

5. $\int_0^1 \int_0^{\sqrt{x}} \frac{2xy}{1-y^4}\ dydx$



$$\int_0^1 \int_0^{\sqrt{x}} \frac{2xy}{1-y^4}\ dydx = \int_0^1 \int_{y^2}^1 \frac{2xy}{1-y^4}\ dxdy = \int_0^1 \frac{x^2y}{1-y^4}\Big|_{y^2}^1\ dy = \int_0^1 \left(\frac{y}{1-y^4} - \frac{y^5}{1-y^4}\right)dy$$

$$= \int_0^1 y\cdot\frac{1-y^4}{1-y^4}\,dy = \int_0^1 y\,dy = \frac{y^2}{2}\Big|_0^1 = \frac{1}{2}$$