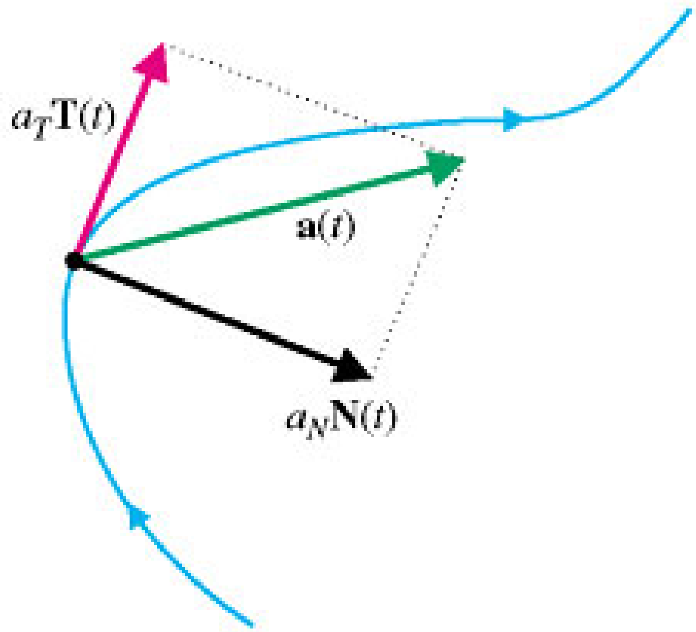
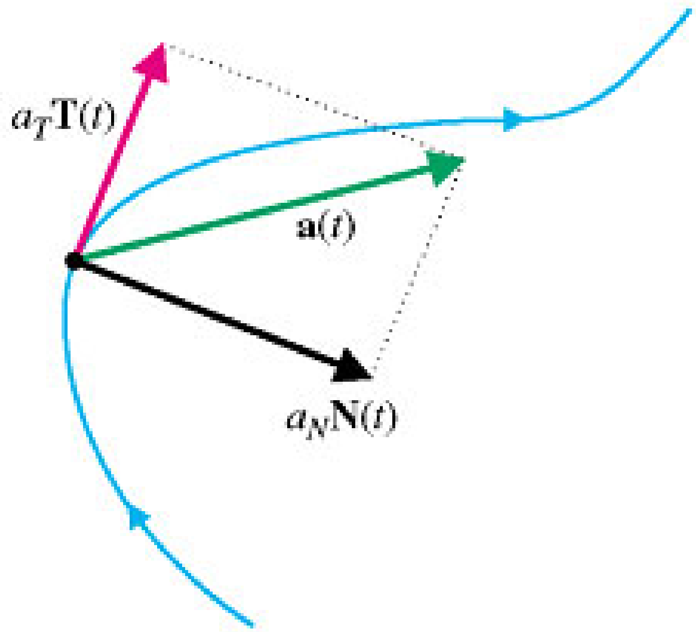
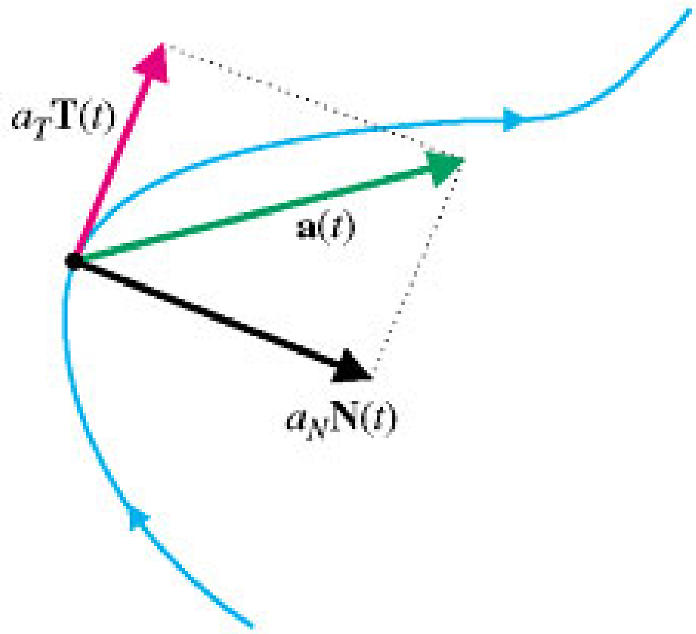
TANGENTIAL AND NORMAL COMPONENTS OF ACCELERATION



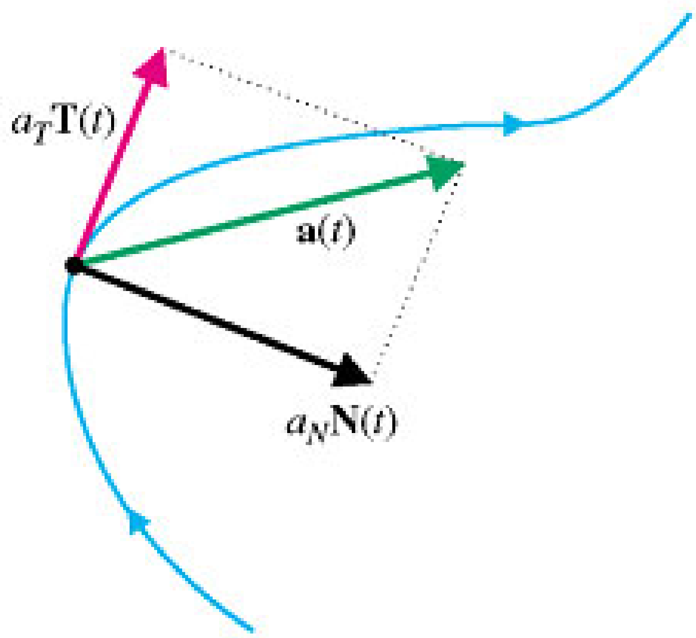
Usually when we have a vector in space, we express that vector in terms of its *i, j,* and *k* components.



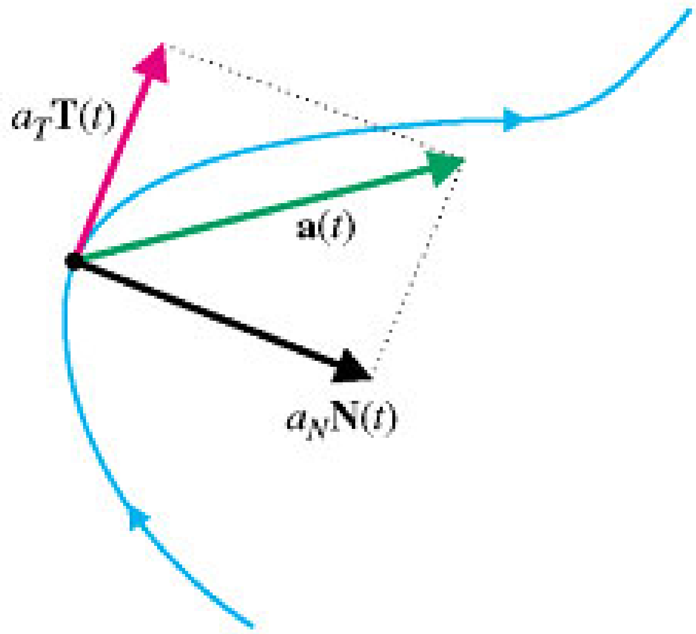
However, in physics it is often better to express an acceleration vector in terms of tangential and normal components.



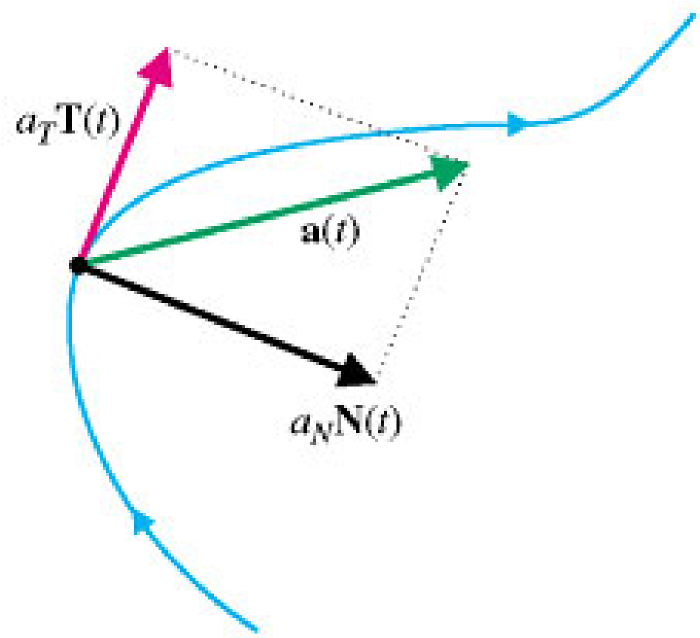
How do we do that?



How do we do that? **Easy!**



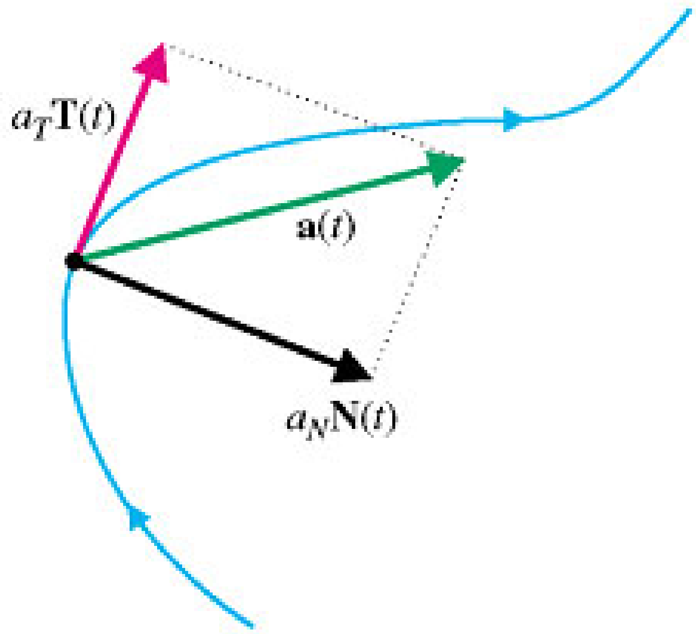
Notice that if $\vec{r}(t)$ is a vector-valued function, then $\vec{v}(t) = \vec{r}'(t) = \left\|\vec{v}(t)\right\| T$.



Notice that if $\vec{r}(t)$ is a vector-valued function, then $\vec{v}(t) = \vec{r}\,'(t) = \|\vec{v}(t)\| T$.

Hence, $\vec{a}(t) = \vec{v}\,'(t) = \left(\|\vec{v}(t)\| T\right)' = \|\vec{v}(t)\|' T + \|\vec{v}(t)\| T'$

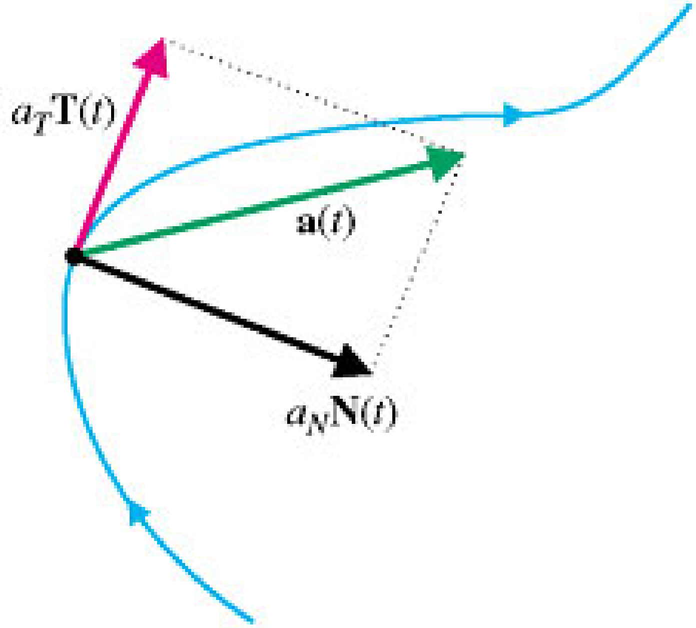
$= \|\vec{v}(t)\|' T + \|\vec{v}(t)\| \|T'\| \dfrac{T'}{\|T'\|} = \|\vec{v}(t)\|' T + \|\vec{v}(t)\| \|T'\| N.$



Thus,

$$\vec{a}_T = \left\|\vec{v}(t)\right\|'$$

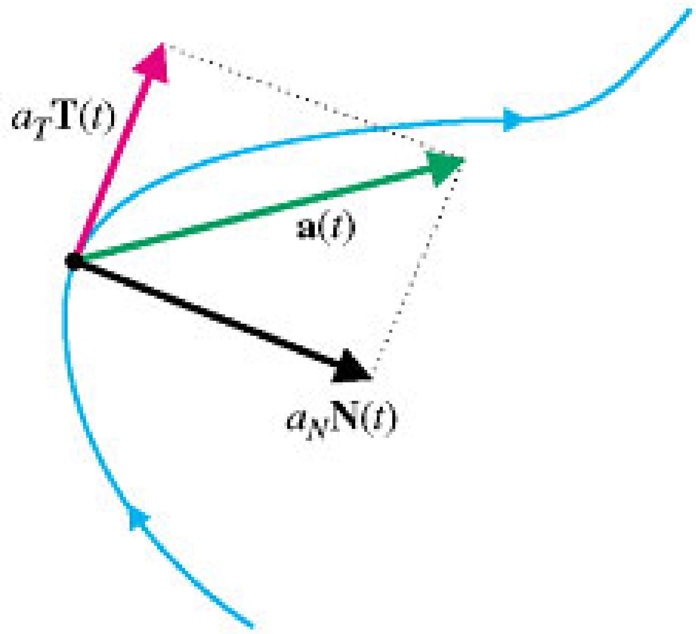
$$\vec{a}_N = \left\|\vec{v}(t)\right\|\left\|T'\right\|$$



However, it's easier to get a_N by using the *Pythagorean Theorem*.

$$\vec{a}_T = \left\|\vec{v}(t)\right\|'$$

$$\vec{a}_N = \sqrt{\left\|\vec{a}(t)\right\|^2 - \left(\vec{a}_T\right)^2}$$

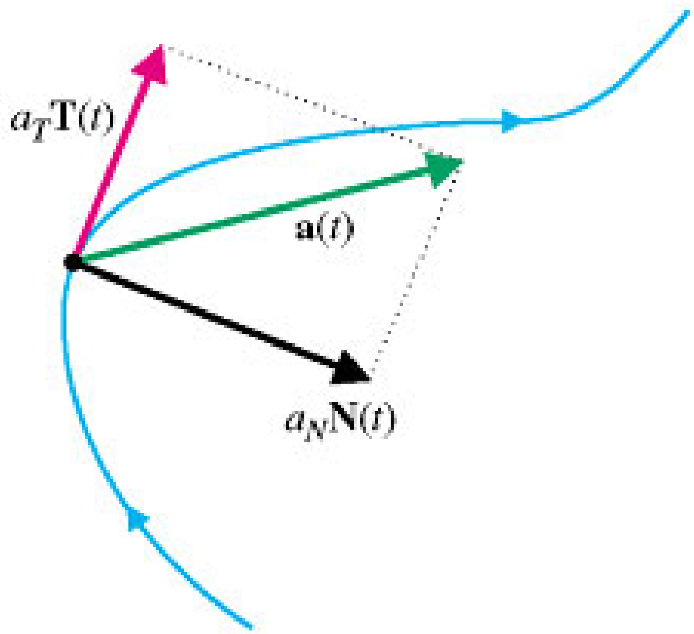


Also, an alternate way to get the tangential component is by taking the dot product of acceleration with the unit tangent vector.

$$\vec{a}_T = \left\|\vec{v}(t)\right\|'$$

$$\vec{a}_T = \vec{a} \bullet T = \vec{a} \bullet \frac{\vec{v}}{\left\|\vec{v}\right\|}$$

$$\vec{a}_N = \sqrt{\left\|\vec{a}(t)\right\|^2 - \left(\vec{a}_T\right)^2}$$



Example:

$$\vec{a}_T = \left\|\vec{v}(t)\right\|'$$

$$\vec{a}_N = \sqrt{\left\|\vec{a}(t)\right\|^2 - \left(\vec{a}_T\right)^2}$$

$$\vec{r}(t) = t^2\,\hat{i} + t\,\hat{j} + t^2\,\hat{k}$$

$$\vec{v}(t) = 2t\,\hat{i} + \hat{j} + 2t\,\hat{k} \qquad \left\|\vec{v}(t)\right\| = \sqrt{4t^2 + 1 + 4t^2} = \sqrt{8t^2 + 1}$$

$$T = \frac{2t}{\sqrt{8t^2+1}}\hat{i} + \frac{1}{\sqrt{8t^2+1}}\hat{j} + \frac{2t}{\sqrt{8t^2+1}}\hat{k}$$

$$\vec{a}(t) = 2\hat{i} + 2\hat{k} \qquad \left\|\vec{a}(t)\right\| = \sqrt{4+4} = \sqrt{8}$$

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$$\vec{a}(t) = 2\hat{i} + 2\hat{k}$$

$$\left\|\vec{a}(t)\right\| = \sqrt{4+4} = \sqrt{8}$$

$$\vec{a}_T = \left\|\vec{v}(t)\right\|' = \frac{1}{2}(8t^2+1)^{-1/2}(16t) = \frac{8t}{\sqrt{8t^2+1}} = \vec{a}\bullet T$$

$$\vec{a}_N = \sqrt{\left\|\vec{a}(t)\right\|^2 - \left(\left\|\vec{v}(t)\right\|'\right)^2} = \sqrt{8 - \frac{64t^2}{8t^2+1}} = \sqrt{\frac{64t^2 + 8 - 64t^2}{8t^2+1}} = \frac{2\sqrt{2}}{\sqrt{8t^2+1}}$$