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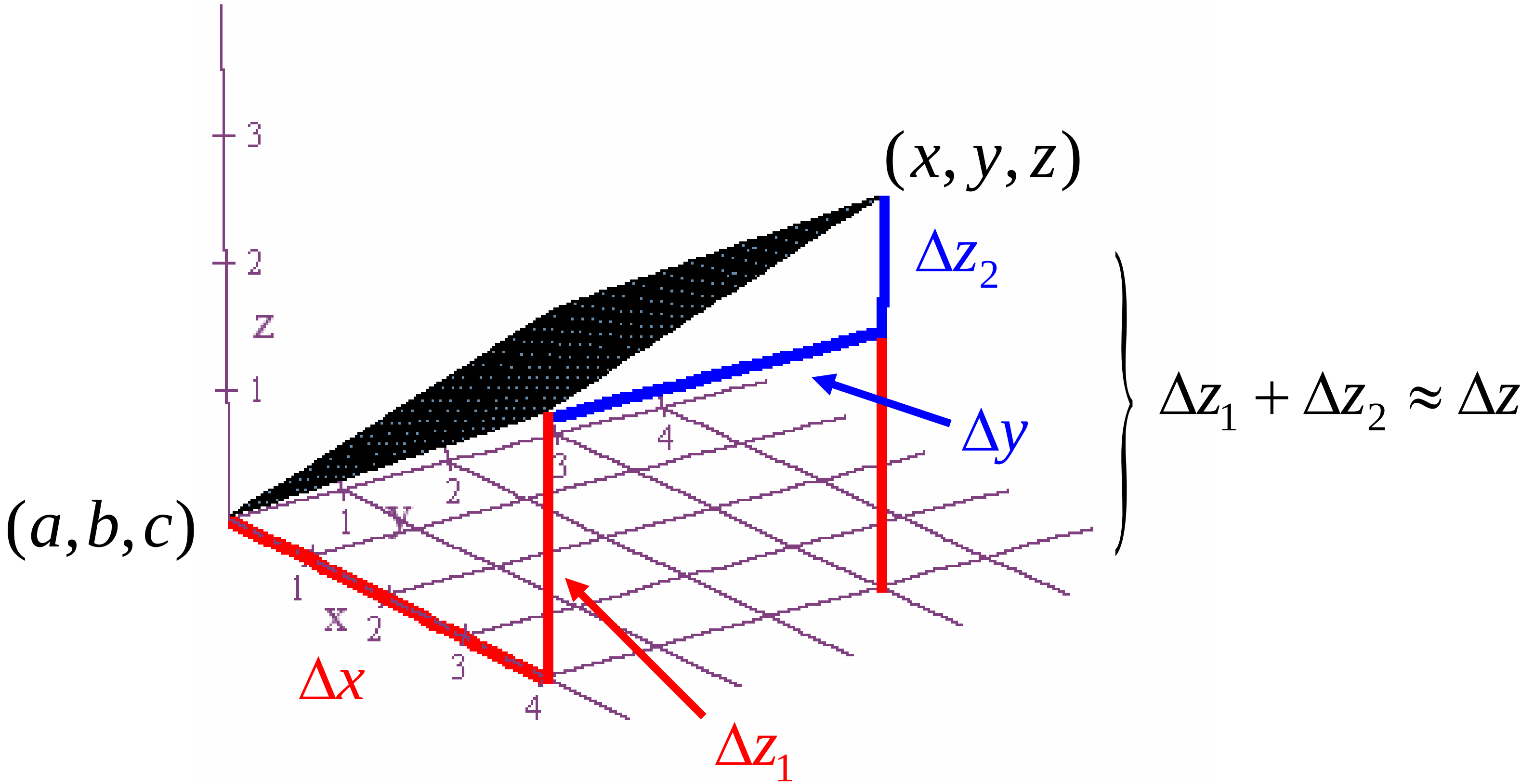
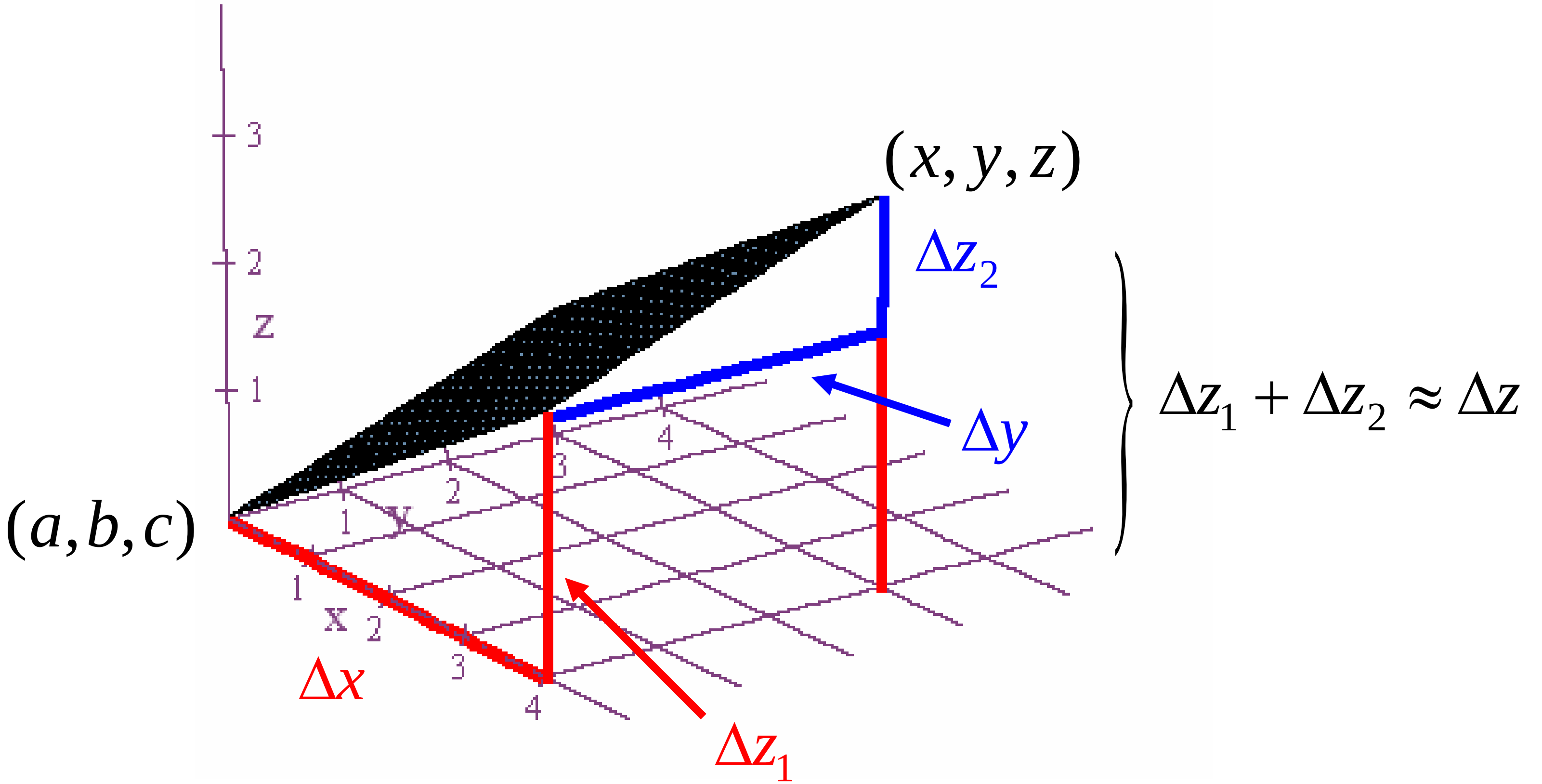
Suppose $z=f(x,y)$ and $x=x(t)$ and $y=y(t)$.

Then the total differential tells us the following:



$$\Delta z \approx \Delta z_1 + \Delta z_2 = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

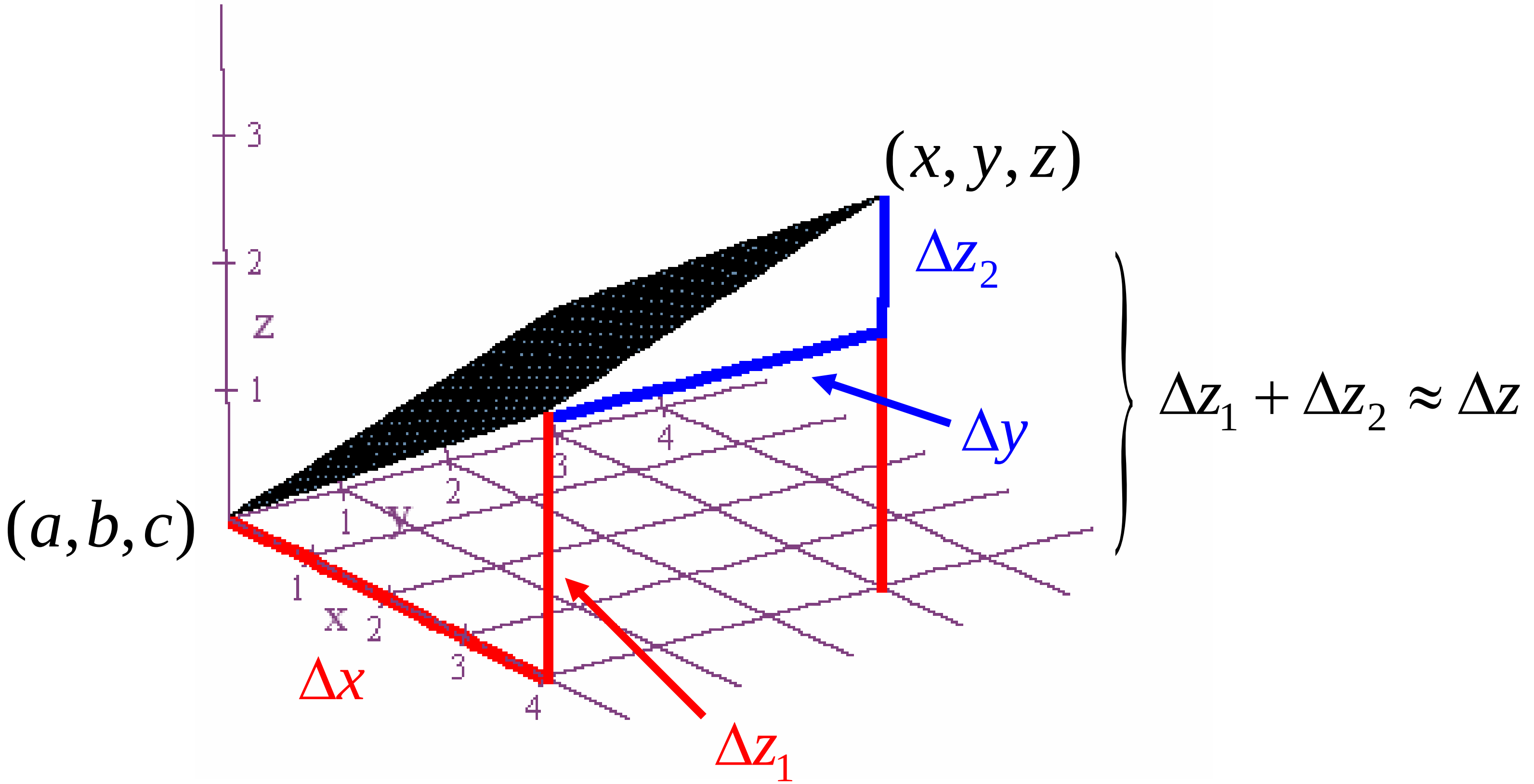
Divide by the change in *t* to get this.



$$\Delta z \approx \Delta z_1 + \Delta z_2 = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

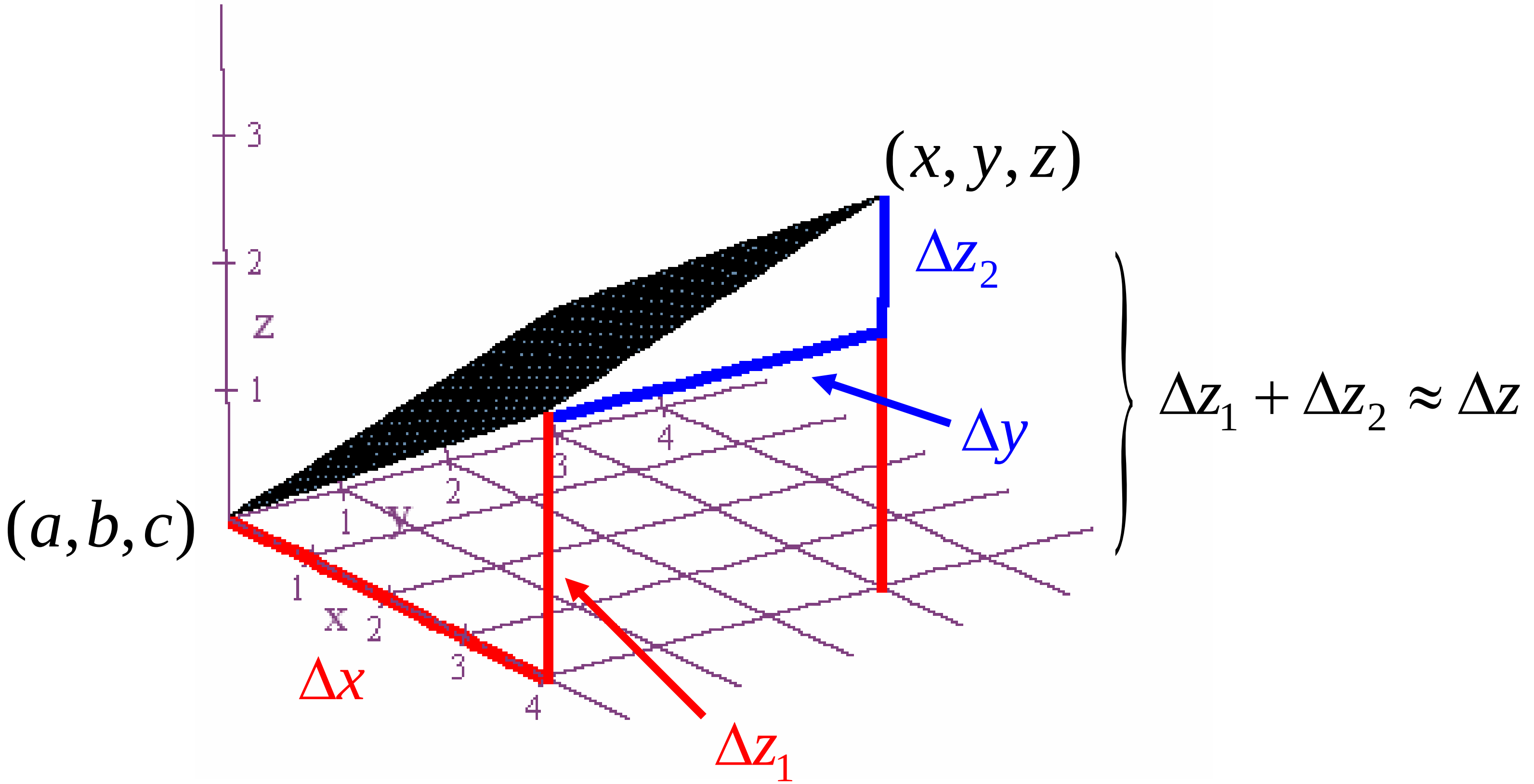
$$\frac{\Delta z}{\Delta t} \approx \frac{\partial z}{\partial x}\frac{\Delta x}{\Delta t} + \frac{\partial z}{\partial y}\frac{\Delta y}{\Delta t}$$

And now, take the limit as *t* goes to zero.



$$\lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t} = \lim_{\Delta t \to 0} \left(\frac{\partial z}{\partial x} \frac{\Delta x}{\Delta t} + \frac{\partial z}{\partial y} \frac{\Delta y}{\Delta t} \right)$$

And you get the Chain Rule!



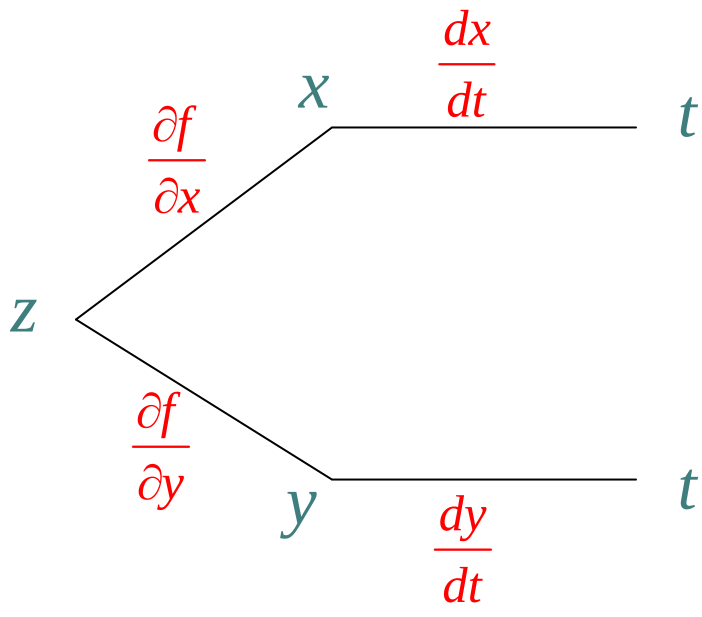
$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Chain Rule Examples

$$z = f(x, y)$$

$$x = x(t)$$

$$y = y(t)$$



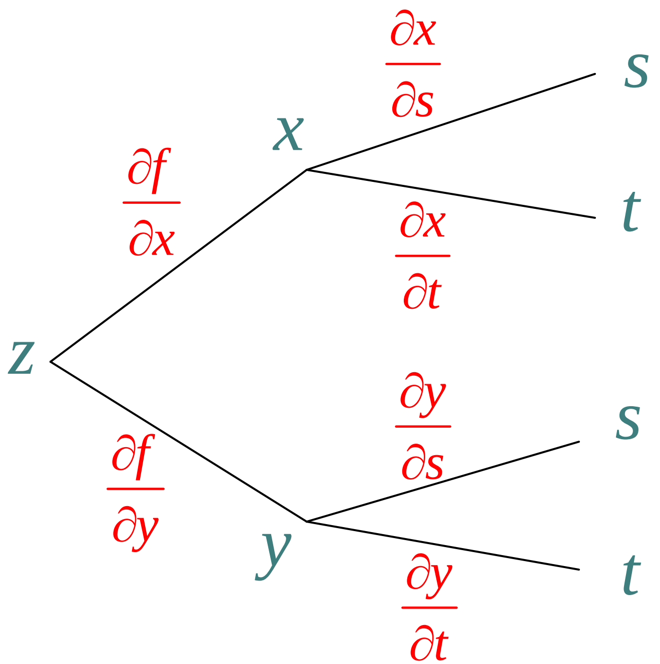
$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Chain Rule Examples

$$z = f(x, y)$$

$$x = x(s,t)$$

$$y = y(s,t)$$



$$\frac{\partial z}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

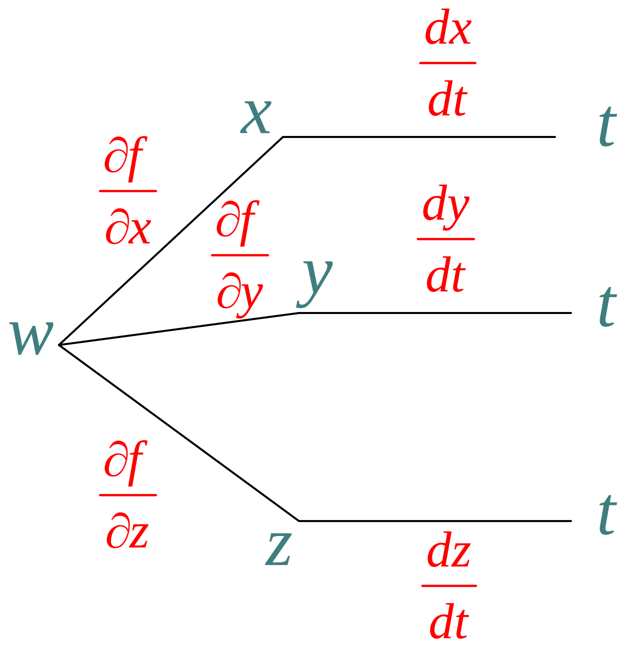
Chain Rule Examples

$$w = f(x, y, z)$$

$$x = x(t)$$

$$y = y(t)$$

$$z = z(t)$$



$$\frac{dw}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$