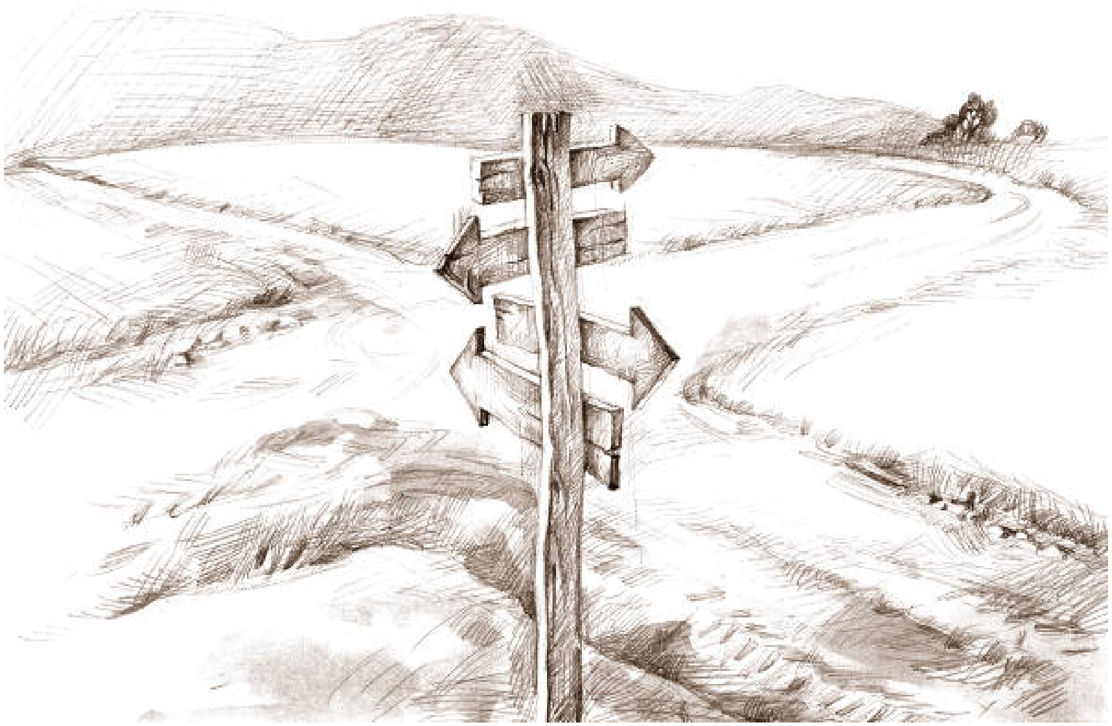
CROSS-SECTIONS



Let's start with the function below and see how we can analyze it without, initially, looking at any graphs.

$$z = x^2 + y^2$$

One thing we can notice immediately is that *z* is nonnegative. Thus, the entire graph must be on or above the *xy*-plane.

$$z = x^2 + y^2$$

Now let's look at cross-sections that we can get by fixing *z* to various values.

$$z = x^2 + y^2$$

$$z = 0 \Rightarrow 0 = x^2 + y^2$$

$$z = 1 \Rightarrow 1 = x^2 + y^2$$

$$z = 2 \Rightarrow 2 = x^2 + y^2$$

$$z = 3 \Rightarrow 3 = x^2 + y^2$$

$$z = 4 \Rightarrow 4 = x^2 + y^2$$

The solution to the first equation is the single point (0,0), but the subsequent equations represent equations for circles.

$$z = x^2 + y^2$$

$$z = 0 \Rightarrow 0 = x^2 + y^2$$

$$z = 1 \Rightarrow 1 = x^2 + y^2$$

$$z = 2 \Rightarrow 2 = x^2 + y^2$$

$$z = 3 \Rightarrow 3 = x^2 + y^2$$

$$z = 4 \Rightarrow 4 = x^2 + y^2$$

Thus, if *z*>0, then a horizontal cross-section of the graph of the function below is going to be a circle.

$$z = x^2 + y^2$$

$$z = 0 \Rightarrow 0 = x^2 + y^2$$

$$z = 1 \Rightarrow 1 = x^2 + y^2$$

$$z = 2 \Rightarrow 2 = x^2 + y^2$$

$$z = 3 \Rightarrow 3 = x^2 + y^2$$

$$z = 4 \Rightarrow 4 = x^2 + y^2$$

Now let's fix some values for *x*.

$$z = x^2 + y^2$$

$$x = -2 \Rightarrow z = 4 + y^2$$

$$x = -1 \Rightarrow z = 1 + y^2$$

$$x = 0 \Rightarrow z = y^2$$

$$x = 1 \Rightarrow z = 1 + y^2$$

$$x = 2 \Rightarrow z = 4 + y^2$$

These are all equations for parabolas that open upward.

$$z = x^2 + y^2$$

$$x = -2 \Rightarrow z = 4 + y^2$$

$$x = -1 \Rightarrow z = 1 + y^2$$

$$x = 0 \Rightarrow z = y^2$$

$$x = 1 \Rightarrow z = 1 + y^2$$

$$x = 2 \Rightarrow z = 4 + y^2$$

Thus, it appears that if we take a cross-section of our surface by fixing an *x* value, then the result is an upward opening parabola.

$$z = x^2 + y^2$$

$$x = -2 \Rightarrow z = 4 + y^2$$

$$x = -1 \Rightarrow z = 1 + y^2$$

$$x = 0 \Rightarrow z = y^2$$

$$x = 1 \Rightarrow z = 1 + y^2$$

$$x = 2 \Rightarrow z = 4 + y^2$$

Finally, let's fix some *y* values.

$$z = x^2 + y^2$$

$$y = -2 \Rightarrow z = x^2 + 4$$

$$y = -1 \Rightarrow z = x^2 + 1$$

$$y = 0 \Rightarrow z = x^2$$

$$y = 1 \Rightarrow z = x^2 + 1$$

$$y = 2 \Rightarrow z = x^2 + 4$$

Again, we get equations for parabolas that open upward.

$$z = x^2 + y^2$$

$$y = -2 \Rightarrow z = x^2 + 4$$

$$y = -1 \Rightarrow z = x^2 + 1$$

$$y = 0 \Rightarrow z = x^2$$

$$y = 1 \Rightarrow z = x^2 + 1$$

$$y = 2 \Rightarrow z = x^2 + 4$$

This is why we are going to call the graph of this surface a *paraboloid*.

$$z = x^2 + y^2$$

$$y = -2 \Rightarrow z = x^2 + 4$$

$$y = -1 \Rightarrow z = x^2 + 1$$

$$y = 0 \Rightarrow z = x^2$$

$$y = 1 \Rightarrow z = x^2 + 1$$

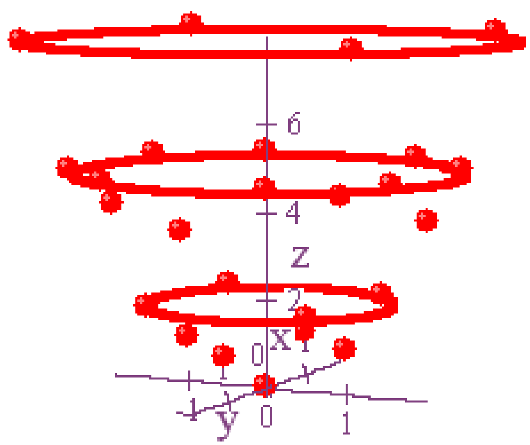
$$y = 2 \Rightarrow z = x^2 + 4$$

Now let's put it all together.

$$z = x^2 + y^2$$

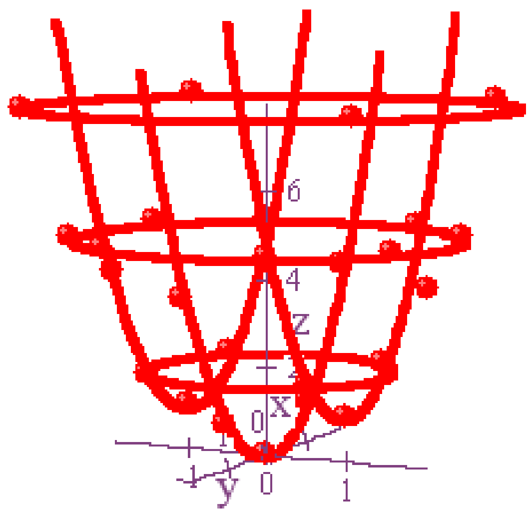
We get (mostly) circular cross-sections by fixing *z*.

$z = x^2 + y^2$



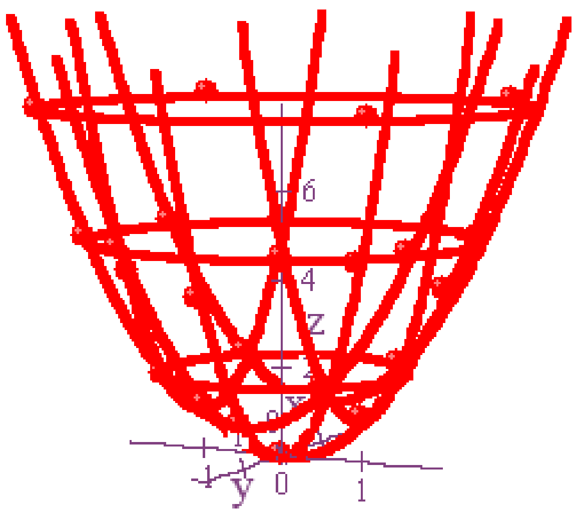
Parabolic cross-sections by fixing *x*.

$$z = x^2 + y^2$$



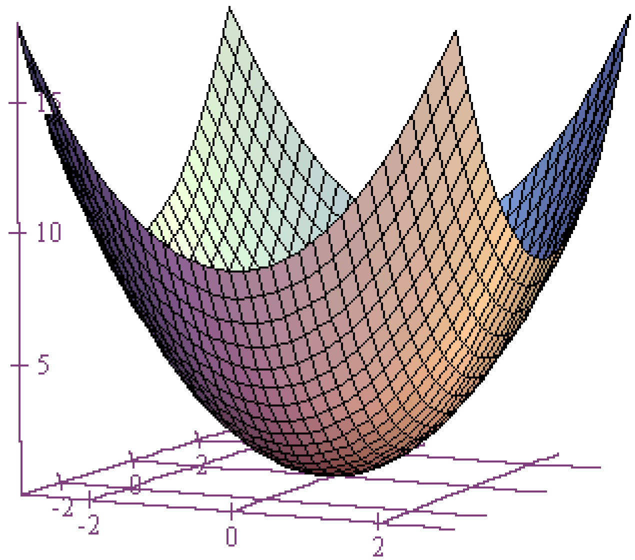
And more parabolic cross-sections by fixing *y*.

$$z = x^2 + y^2$$



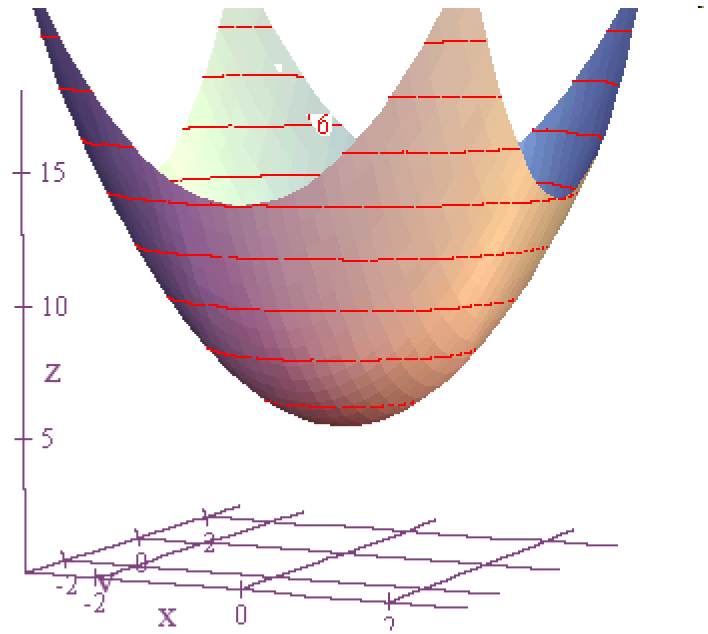
These cross-sections give us a pretty good idea of what the graph of our surface looks like.

$$z = x^2 + y^2$$



Now one more thing. If we graph the cross-sections obtained by fixing *z* on the surface itself, then we often call those cross-sections *contours*.

$$z = x^2 + y^2$$



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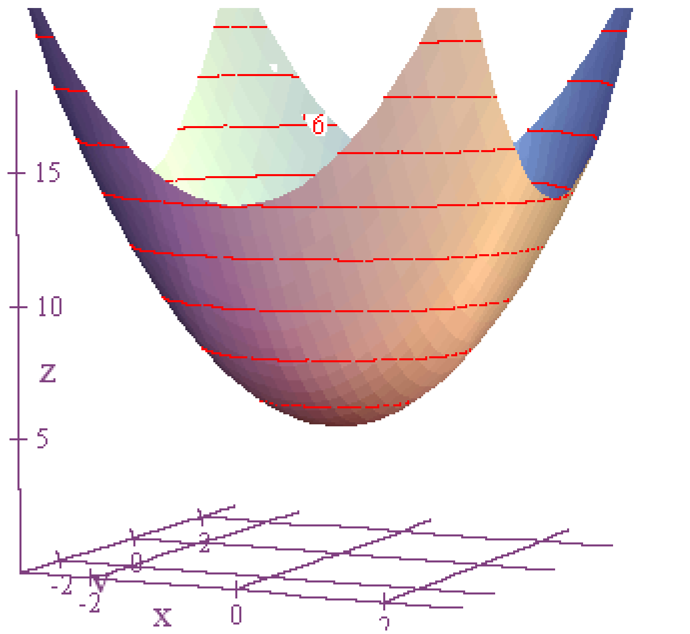
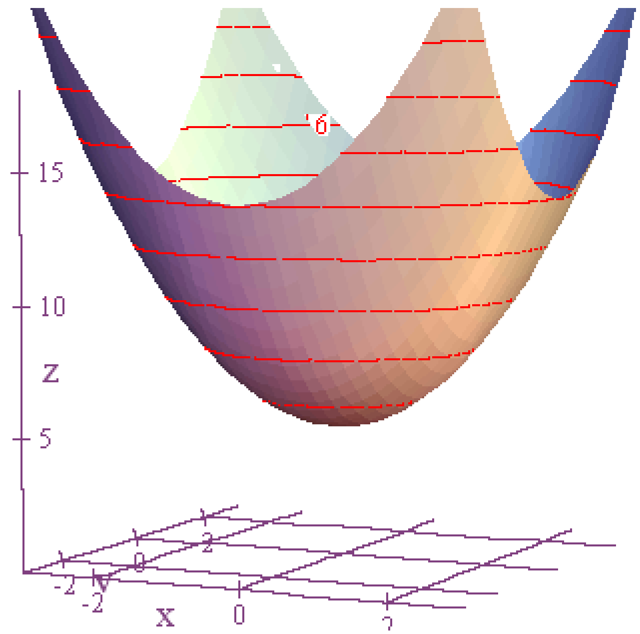
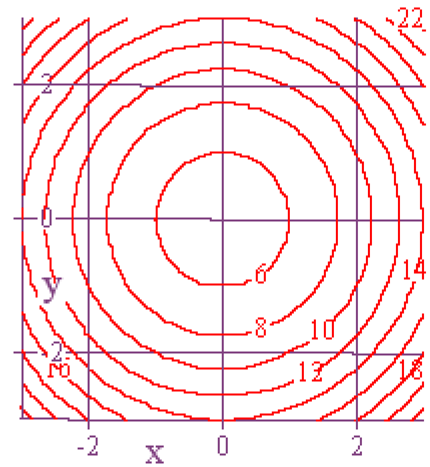


However, if we graph these same cross-sections in just the *xy*-plane, then we call them *level curves*.

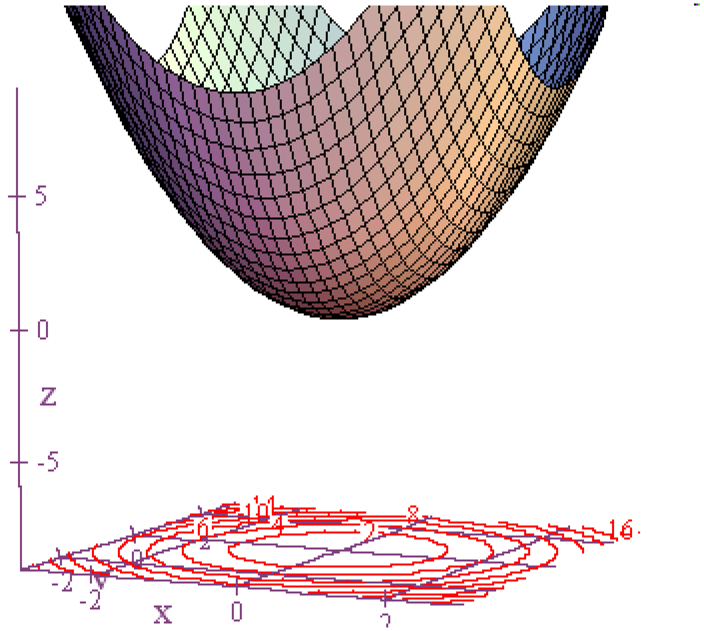
$$z = x^2 + y^2$$





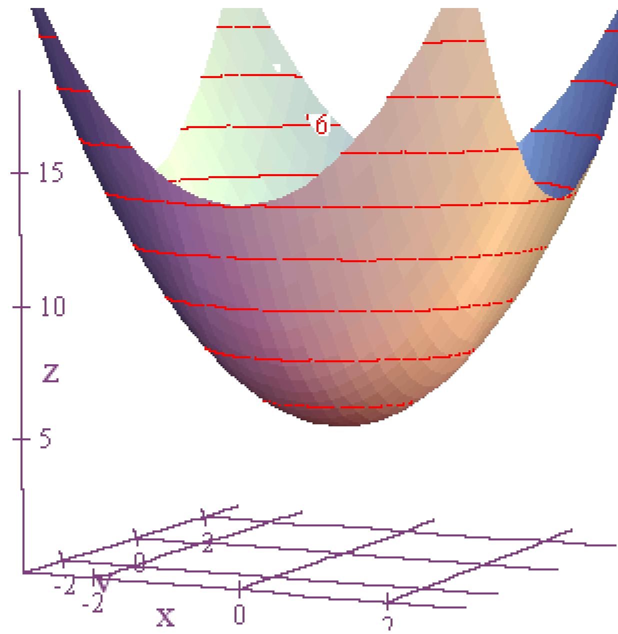
Here is a graph of our surface with the level curves down below.

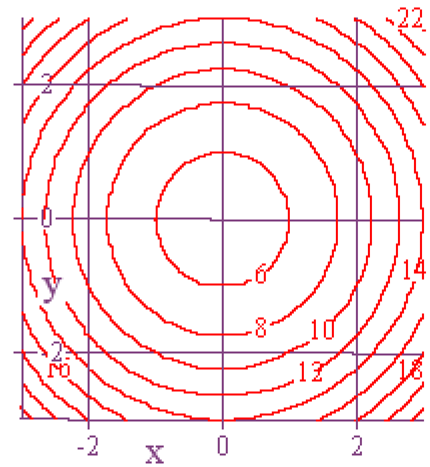
$$z = x^2 + y^2$$



CAUTION: Some people use the term *contours* for both situations, and others use the term *level curve* for both.

$$z = x^2 + y^2$$





Either way, cross-sections help us analyze the graph of a surface.

$$z = x^2 + y^2$$