DOUBLE INTEGRALS

$$\iint_R dA$$

Suppose we have a function $z = f(x, y)$ defined on a region R in the xy-plane, and just to make life a little easier, let's suppose that $z > 0$. Then we can approximate the volume between z and R by subdividing the region R into subrectangles with sides Δx and Δy and evaluating our function at an arbitrary point in each rectangle in order to get a value to use as height. We could then find the volume of a box erected over each subrectangle and make the following approximation of the volume.

$$\text{Volume} \approx \sum_{i,j} f(x_{ij}, y_{ij}) \Delta x \Delta y$$

To get the exact volume we merely take a limit as $\Delta x, \Delta y \to 0$.

$$\text{Volume} = \lim_{\Delta x, \Delta y \to 0} \sum_{i,j} f(x_{ij}, y_{ij}) \Delta x \Delta y = \iint_R f(x, y)\, dA$$

To get the exact volume we merely take a limit as $\Delta x, \Delta y \rightarrow 0$.

$$\text{Volume} = \lim_{\Delta x, \Delta y \to 0} \sum_{i,j} f(x_{ij}, y_{ij}) \Delta x \Delta y = \iint_R f(x, y)\, dA$$

We call this result a double integral, and most people today use two integral signs to denote it. However, our book uses an older notation involving just a single integral sign, $\int_R f(x, y)\, dA$.

THEOREM: If $z = f(x, y)$ is continuous on a rectangular region R and if we partition R into a series of subrectangles with sides Δx and Δy, then the double integral of $z = f(x, y)$ over R is defined by

$$\iint_R f(x, y)\, dA = \lim_{\Delta x, \Delta y \to 0} \sum_{i,j} f(x_{ij}, y_{ij}) \Delta x \Delta y$$