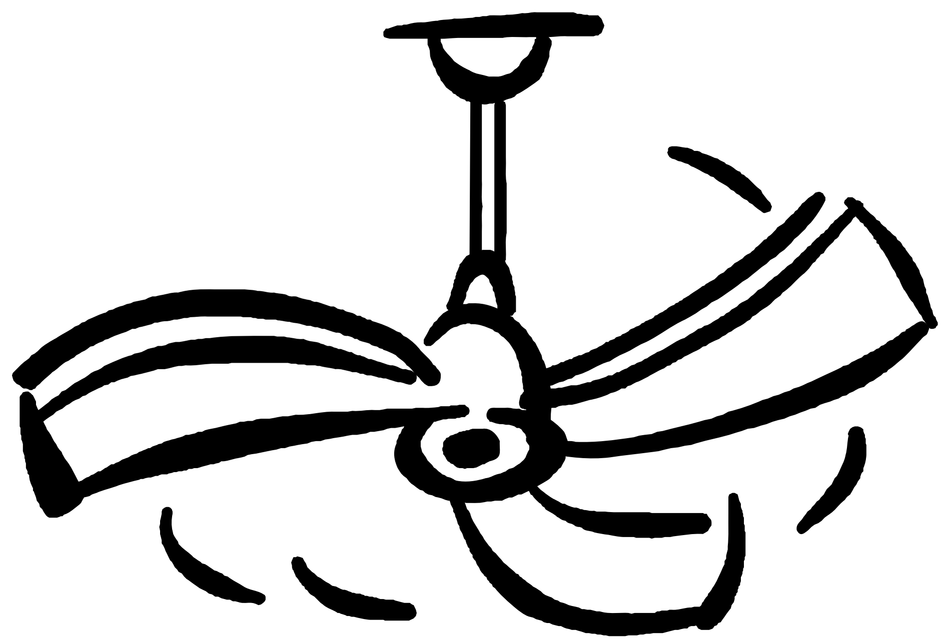
# EXTRA STOKES

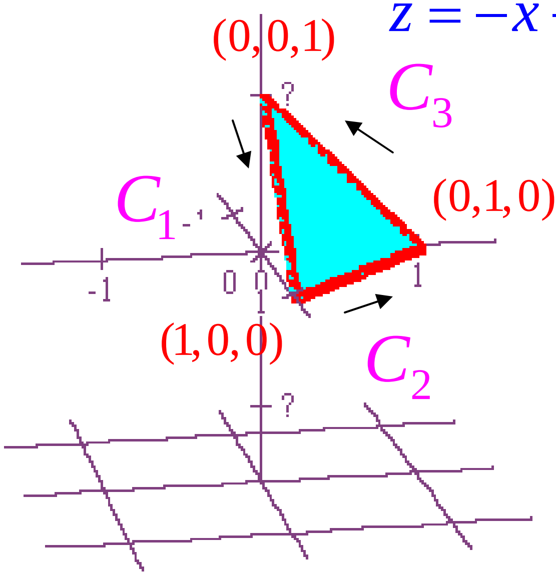


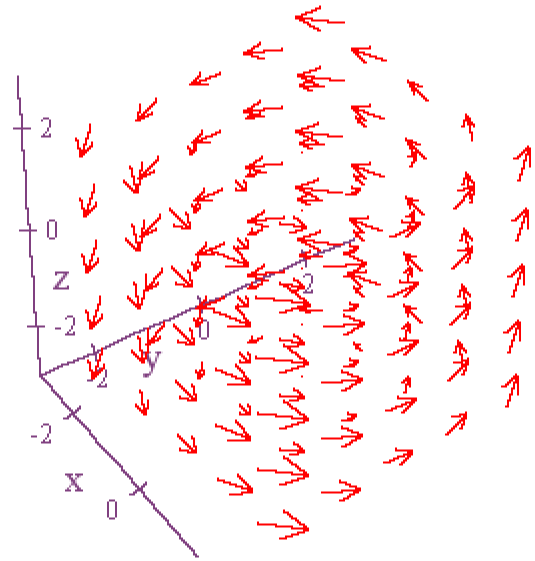
One can never do too much work!

$$\vec{F} = -y\,\hat{i} + x\,\hat{j}$$

$$C = C_1 + C_2 + C_3$$

$$z = -x - y + 1$$



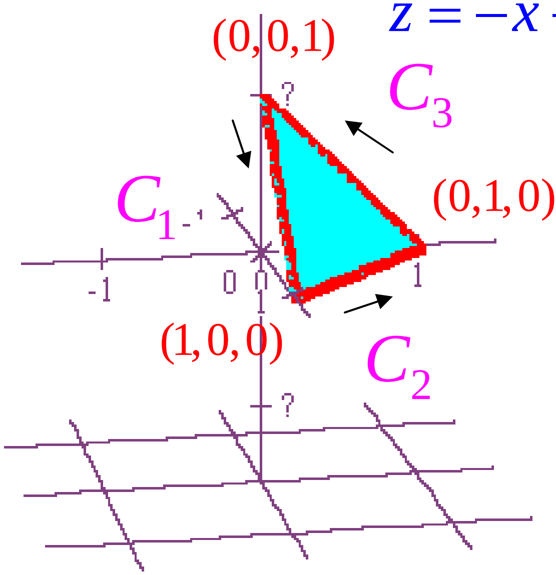



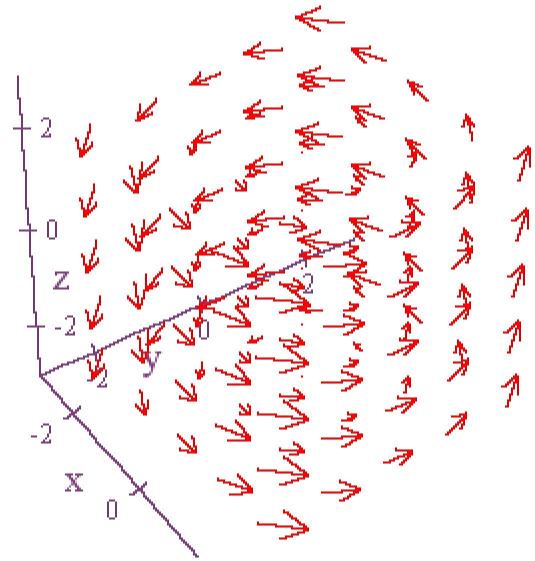
We could find the work done by the vector field by evaluating it along each path.

$$\vec{F} = -y\,\hat{i} + x\,\hat{j}$$

$$C = C_1 + C_2 + C_3$$

$$z = -x - y + 1$$



We could find the work done by the vector field by evaluating it along each path.

$$\vec{F} = -y\hat{i} + x\hat{j}$$

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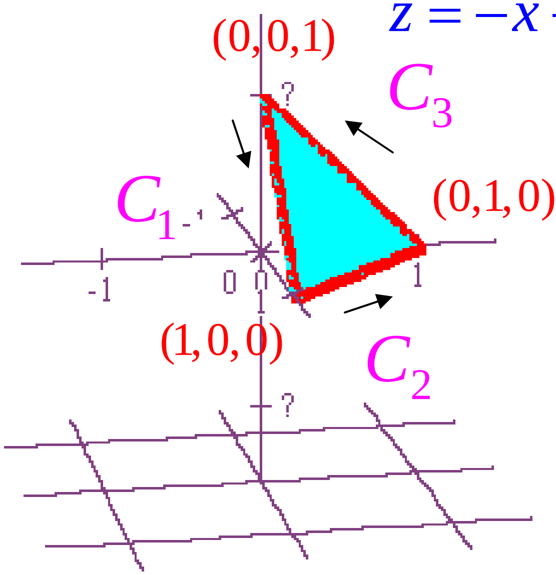



$$C_1: \quad x = t \qquad dx/dt = 1$$
$$y = 0 \qquad dy/dt = 0$$
$$z = 1 - t \qquad dz/dt = -1$$
$$0 \le t \le 1$$

$$\int_{C_1} \vec{F} \bullet d\vec{r} = \int_{C_1} P\,dx + Q\,dy + R\,dz$$

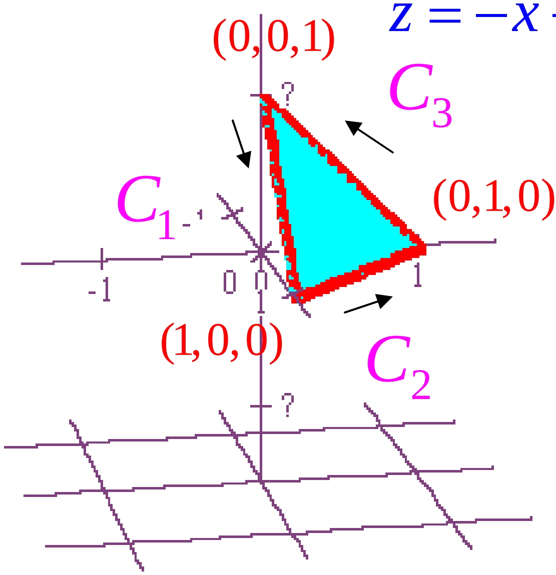
$$= \int_0^1 (-0 \cdot 1 + t \cdot 0 + 0 \cdot -1)\,dt = 0$$

We could find the work done by the vector field by evaluating it along each path.

$\vec{F} = -y\hat{i} + x\hat{j}$

$C = C_1 + C_2 + C_3$

$z = -x - y + 1$




$C_2: \quad x = 1 - t \quad dx/dt = -1$

$y = t \quad dy/dt = 1$

$z = 0 \quad dz/dt = 0$

$0 \le t \le 1$

$$\int_{C_2} \vec{F} \bullet d\vec{r} = \int_{C_2} P\,dx + Q\,dy + R\,dz$$

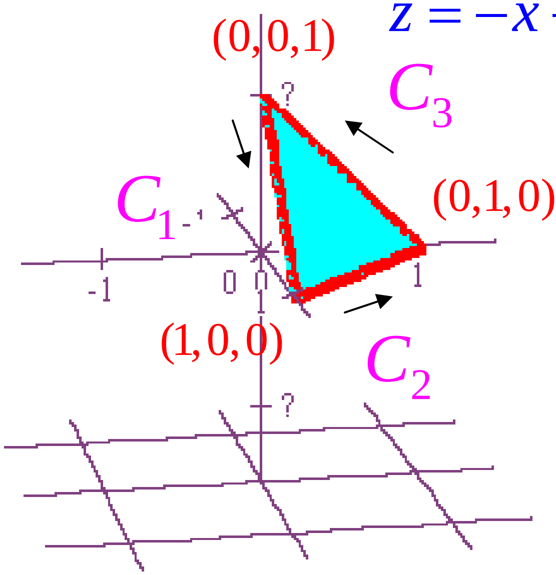
$$= \int_0^1 (-t \cdot -1 + [1 - t] \cdot 1 + 0 \cdot 0)\,dt = 1$$

We could find the work done by the vector field by evaluating it along each path.

$\vec{F} = -y\hat{i} + x\hat{j}$

$C = C_1 + C_2 + C_3$

$z = -x - y + 1$




$$C_3: \quad x = 0 \qquad dx/dt = 0$$
$$y = 1 - t \qquad dy/dt = -1$$
$$z = t \qquad dz/dt = 1$$
$$0 \le t \le 1$$

$$\int_{C_3} \vec{F} \bullet d\vec{r} = \int_{C_3} Pdx + Qdy + Rdz$$

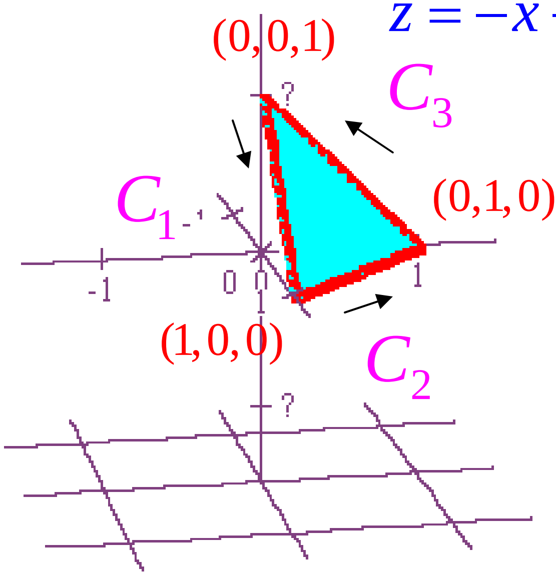
$$= \int_0^1 ([-1+t]\cdot 0 + 0 \cdot -1 + 0 \cdot 1)\, dt = 0$$

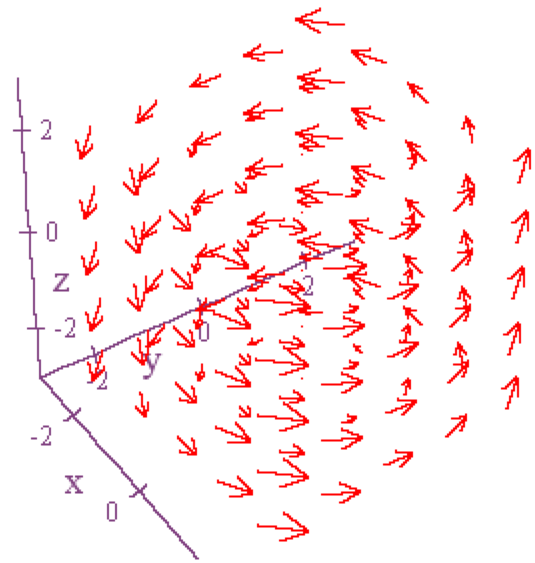
We could also evaluate the work done by the vector field by evaluating it along each path.

$$\vec{F} = -y\,\hat{i} + x\,\hat{j}$$

$$C = C_1 + C_2 + C_3$$

$$z = -x - y + 1$$



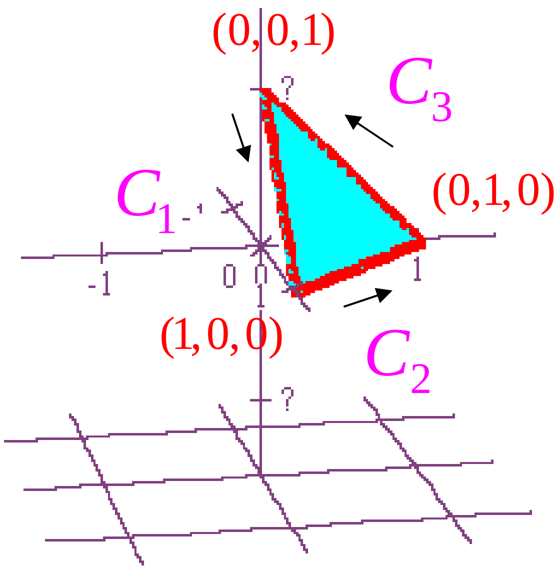

Therefore,

$$\int_C \vec{F} \bullet d\vec{r} = \int_{C_1} \vec{F} \bullet d\vec{r} + \int_{C_2} \vec{F} \bullet d\vec{r} + \int_{C_3} \vec{F} \bullet d\vec{r} = 0 + 1 + 0 = 1$$

We could also find the work done by using our more familiar version of Stokes' Theorem.

$\vec{F} = -y\hat{i} + x\hat{j}$

$C = C_1 + C_2 + C_3$




$z = -x - y + 1$

$g = x + y + z$

$\nabla g = \hat{i} + \hat{j} + \hat{k}$

$\nabla \times \vec{F} = 2\hat{k}$

$$\int_C \vec{F} \bullet d\vec{r} = \iint_R (\nabla \times \vec{F}) \bullet \nabla g \, dA = \iint_R (2\hat{k}) \bullet (\hat{i} + \hat{j} + \hat{k}) \, dA = \iint_R 2 \, dA = 2 \cdot \frac{1}{2} = 1$$