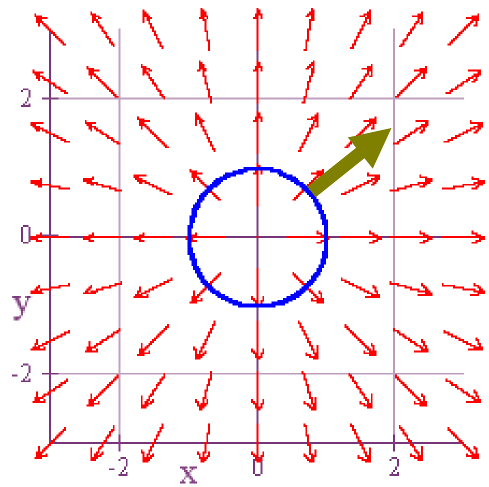
DIVERGENCE THEOREM IN HIGHER DIMENSIONS

To find the flux across a closed, counterclockwise oriented plane curve caused by a vector field *F*=<*P*,*Q*>, we used the following formula along with an outward pointing unit normal vector *N*.

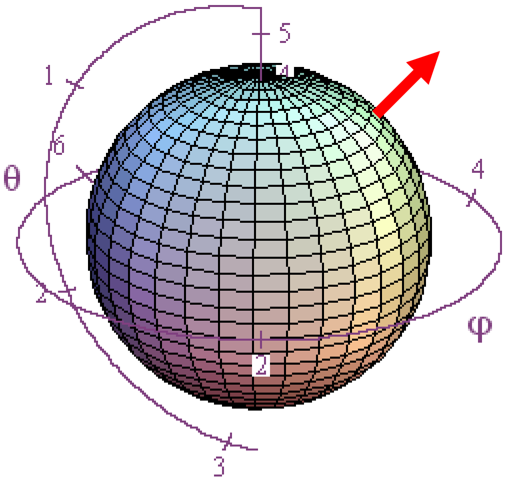
$$\text{Flux} = \int_C F \bullet N \, ds = \iint_R div\, F \, dA = \iint_R \left(\nabla \bullet F \right) dA = \iint_R \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} \right) dA$$



In higher dimensions, we have a similar formula involving an outward pointing unit normal vector *N*.

$$\text{2d-Flux } = \int_C F \bullet N \, ds = \iint_R div\, F \, dA = \iint_R (\nabla \bullet F) \, dA$$

$$\text{3d-Flux } = \iint_S F \bullet N \, dS = \iiint_V div\, F \, dV = \iiint_V \nabla \bullet F \, dV$$

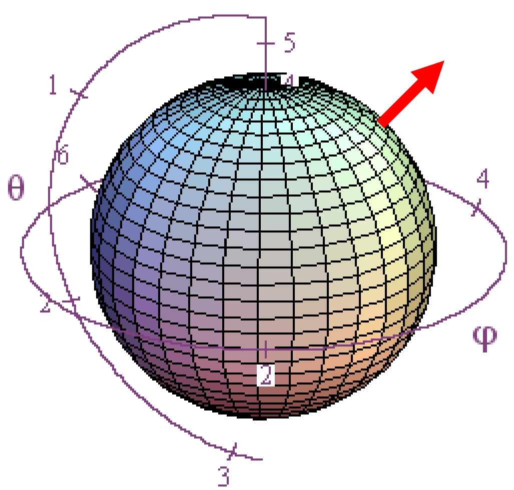


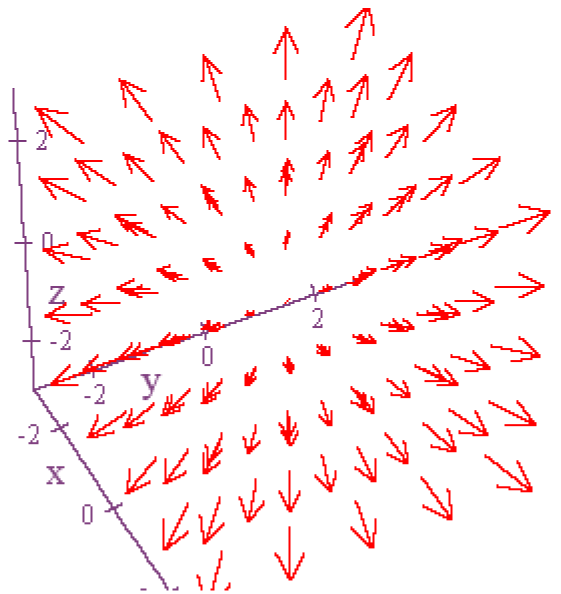
Let’s do a problem!

$$\iint_S F \bullet N \, dS = \iiint_V div\, F \, dV = \iiint_V \nabla \bullet F \, dV$$

V is the solid ball with surface S defned by $x^2 + y^2 + z^2 = 4$

$F = x\hat{i} + y\,\hat{j} + z\,\hat{k}$





Just integrate!

$$\iint_S F \bullet N\, dS = \iiint_V div\, F\, dV = \iiint_V \nabla \bullet F\, dV$$

V is the solid ball with surface S defned by $x^2 + y^2 + z^2 = 4$

$F = x\hat{i} + y\,\hat{j} + z\,\hat{k}$

$$\iint_S F \bullet N dS = \iiint_V \nabla \bullet F\, dV = \iiint_V (1+1+1)\, dV = 3\iiint_V dV = 3 \cdot \frac{4\pi}{3} \cdot 2^3 = 32\pi$$

It's so easy, a highly intelligent caveman could do it!

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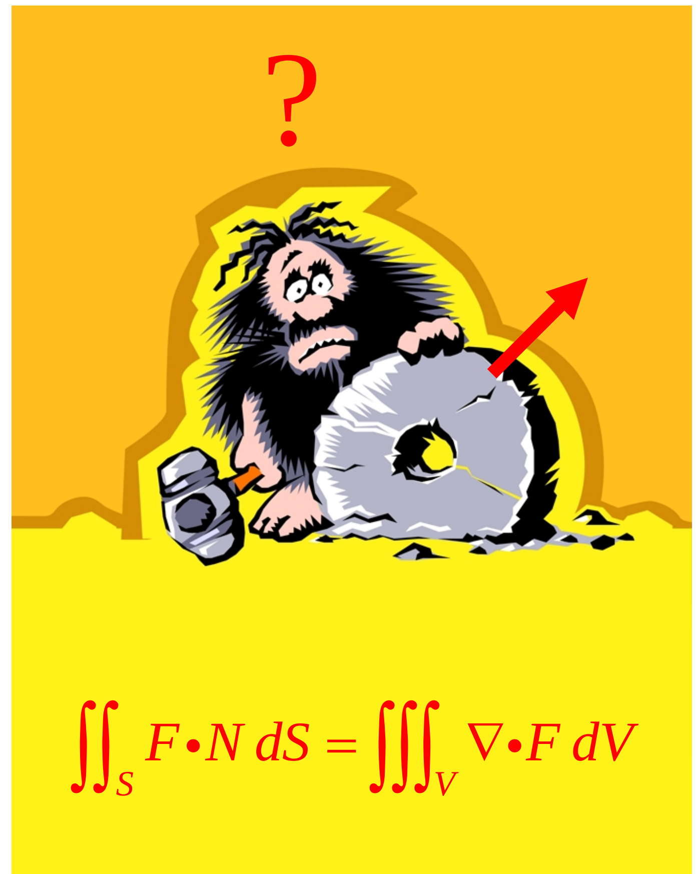

$$\iint_S F \bullet N dS = \iiint_V \nabla \bullet F\, dV = \iiint_V (1+1+1)\, dV = 3\iiint_V dV = 3 \cdot \frac{4\pi}{3} \cdot 2^3 = 32\pi$$